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LABORATORY TESTS OF REINFORCED CONCRETE ARCH RIBS

BY

WILBUR M. WILSON

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ENGINEERING EXPERIMENT STATION

BULLETIN No. 202 FEBRUARY, 1930

LABORATORY TESTS OF REINFORCED CONCRETE ARCH RIBS

BY

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ENGINEERING EXPERIMENT STATION

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LABORATORY TESTS OF REINFORCED CONCRETE ARCH RIBS

I. Introduction

1. *Object of Investigation.*—The arch has always been a favorite type of structure. The development of highways and the increased use of concrete has made the use of reinforced concrete arches very common and the annual investment in this type of structure is growing rapidly. The arch deserves its popularity for, when properly designed, it is one of our most beautiful structures and, in certain situations that are common, it is one of the most economical types of bridge supports. Any additional information that will result in safer or more economical design of a structure that is so largely used would seem to be desirable.

One object of the investigation was to compare measured stresses with stresses computed by the elastic theory, both when the stresses are produced by loads and when they are produced by abutment movements. The theory itself, being a proposition in geometry, needs no experimental verification, but it can only be applied to an arch by making certain assumptions relative to the behavior of the material. The validity of using the elastic theory as a means of analyzing an arch depends upon the magnitude of the errors introduced in the analysis because of the errors in the assumptions.

The strength of concrete is usually determined by tests of 6-in. by 12-in. cylinders subjected to centric axial forces, whereas the rib of an arch is subjected to axial compression, transverse shear, and flexure. Moreover, the ratio of length to width is many times greater for an arch rib than for a standard control cylinder. One object of the tests was to compare the strength of concrete developed in an arch with the strength of the same concrete as developed in a 6-in. by 12-in. cylinder.

Arch ribs are long and slender, the ratio of unsupported length to width sometimes being greater for arches than the permissible value of the same ratio for columns. Arches of various widths were tested in order that the effect of the slenderness-ratio of a rib upon its strength might be determined.

Spiral reinforcement is known to increase the strength of concrete columns. A few tests were made to determine whether spiral reinforcement likewise increases the strength of arch ribs.

The magnitude of the deformation of concrete is known to depend upon the length of time that the load is applied. Tests were made to determine the effect of time yield upon the relation between reactions and abutment displacements.

2. *Acknowledgments.*—This investigation is a part of the work of the Engineering Experiment Station of the University of Illinois, of which DEAN MILO S. KETCHUM is the director, and of the Department of Civil Engineering, of which PROF. W. C. HUNTINGTON is the head. It also constitutes a part of the program of the Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers.*

The experimental work was done by Cyrus Fishburn, F. T. Mavis, and R. G. Sturm, technical assistants† employed by the Arch Committee, and by A. H. Sorenson and E. C. Grafton, Research Graduate Assistants† of the Engineering Experiment Station, all working under the supervision of the author. The direct expenses of the tests were paid from funds provided by the AMERICAN SOCIETY OF CIVIL ENGINEERS, and the ENGINEERING FOUNDATION. The gravel and sand used in making the specimens were contributed by the NEAL GRAVEL COMPANY, and the cement was contributed by the PORTLAND CEMENT ASSOCIATION.

The writer acknowledges his indebtedness to HARDY CROSS, Professor of Structural Engineering, for many valuable conversations pertaining to the investigation.

II. DESCRIPTION OF SPECIMENS AND APPARATUS

3. *Description of Specimens.*—The specimens used in this investigation were reinforced concrete, hingeless, arch ribs having a span of 17 ft. 6 in. and a rise of 4 ft. $\frac{1}{16}$ in., measured to points on the arch axis. The intrados and extrados were circular arcs having radii of 10 ft. $8\frac{1}{4}$ in. and 12 ft. 6 in., respectively, centered to give a depth at the crown of 5 in., and at the springing of about 10.8 in. The arch rib is shown in elevation in Fig. 1.

The four arches used in the study to determine the effect of the slenderness ratio of an arch upon its strength were arches 26-1, 26-2, 26-3, and 26-4, which had widths of 8 in., $6\frac{1}{8}$ in., $4\frac{1}{2}$ in., and $3\frac{1}{8}$ in., respectively. Each rib was reinforced with four plain round rods placed one in each corner of a section, $\frac{3}{4}$ in. from the intrados and

*The members of this Committee are C. T. Morris, Chairman, E. H. Harder, A. C. Janni, George E. Beggs, and W. M. Wilson.

†These Assistantships were financed by the American Society of Civil Engineers.

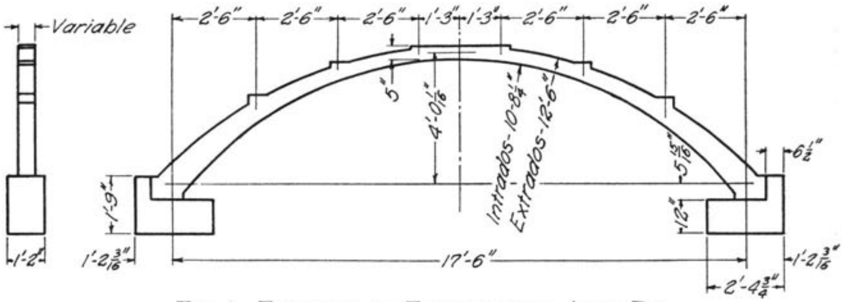


FIG. 1. ELEVATION OF EXPERIMENTAL ARCH RIB

extrados respectively, and ¾ in. from the vertical faces, all distances being measured from the centers of the rods. The diameters of the rods for the four ribs were ½ in., 7/16 in., ⅜ in., and 5/16 in., respectively. This combination of width of rib and diameter of rod made the percentage of steel the same in all ribs, namely, 1.96 per cent at the crown.

Each of the other arches was 6⅛ in. wide and was reinforced with four ½-in. round rods located as described in the previous paragraph. In addition, four of these arches were reinforced with spirals made of ¼-in. rounds, and having a pitch of 1 in. The section at the crown was small, and the spiral was inside of the longitudinal bars, and therefore did not enclose a very large proportion of the total rib.

The concrete abutments, 14 in. wide normal to the rib, 28¾ in. long, and 21 in. deep, were cast integrally with the rib. A 1-in. round rod extending horizontally through each abutment near its center was provided for attaching lifting devices when the arch was moved.

Two ½-in. round rods cast in and connecting the two abutments served a dual purpose, in that they took the horizontal dead load thrust while the arch was being moved and, when sawed in two near one abutment, they served as measuring rods to control the length of the span during the tests.

The forms for the arches are shown in Fig. 2. The sides were cast-iron channel sections, the inside surfaces and the flanges being machined. The bottom was a curved structural steel plate ¼ in. thick, bolted to the bottom flanges of the side pieces. The top was composed of a number of plates about one foot long bolted to the top flanges; the top plates were added, beginning at the ends, as the arch was poured. The same forms were used for making arches of various widths, the top and bottom plates having a line of holes

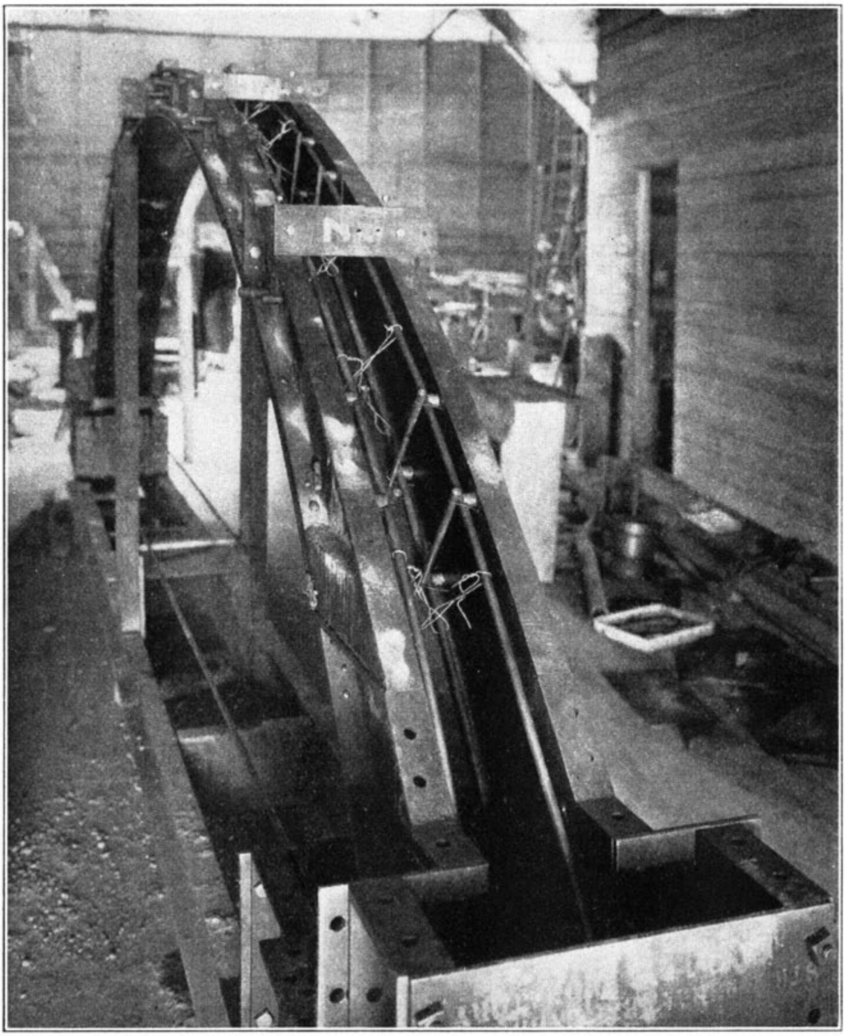


FIG. 2. FORMS FOR ARCHES

properly located for each width of arch desired. The holes in the top and bottom plates were spotted after the sides had been carefully aligned with a transit, and the forms proved to be very true. From the nature of their construction they could not be varied from the original size and alignment once the holes in the plates and flanges had been drilled.

TABLE 1

SIEVE ANALYSIS OF AGGREGATES

Size of Sieve	Per Cent Not Passing Screen	
	Fine Aggregate	Coarse Aggregate
1.5	0.0	0.0
0.75	0.0	0.0
0.375	0.0	43.2
No. 4	4.4	99.1
No. 8	22.4	99.5
No. 14	44.1	100.0
No. 28	71.2	100.0
No. 48	92.0	100.0
No. 100	97.4	100.0
Fineness Modulus	3.32	6.42

The reinforcing bars, shown in the forms in Fig. 2, were held in place by means of stools and radial rods. The stools, made of ¾-in. conduit, performed two functions: they helped to hold the reinforcing in place, and they provided a hole from the surface of the concrete to the steel, making it possible to measure the strain in the steel with a strain gage. The stools were set over metal buttons bolted to the forms and could be placed quickly and accurately. They could not move out of their correct position after having once been set. The radial rods, ½-in. rounds, also performed two functions; in addition to holding the reinforcing rods in place they served as strain-gage plugs for measuring the strain in the concrete.

The concrete proportions by volume were one part Portland cement, 2.8 parts sand, 2.8 parts gravel, and 1.1 parts water. The arches were small, making it necessary to use a comparatively fine gravel; this fact accounts for the small amount of gravel that was used. A typical sieve analysis of the aggregates is given in Table 1.

Although the concrete was a 1:2.8:2.8 mix by volume, the actual quantities used were determined by weight, the weight of the aggregate and the quantity of water to be added being corrected for the amount of water contained in the aggregate as determined by moisture tests made just previous to the pouring of each arch. The weight of the materials that composed a batch, based on dry aggregate, was as follows:

Cement	25.0	Pounds
Sand	80.5	"
Gravel	73.0	"
Water	17.2	"

Three 6-in. by 12-in. control cylinders were poured for each arch. The arches and cylinders were stripped when two days old and wrapped in burlap. The burlap wrapping was thoroughly soaked with water every other day for a period of two weeks, then the burlap was removed from both the arch and the cylinders. The cylinders and arches were stored in the laboratory until they were tested at an age of approximately 30 days for arches 26-1 to 26-11, and approximately 60 days for arches 27-3 and 27-4.

4. *Description of Apparatus.*—The arches were tested in a 300 000-pound two-screw Riehle testing machine, having wings with an overall length of about 21 ft. Figure 3 shows an arch in the machine ready to be tested.

The loading mechanism shown in Fig. 4 was used in testing arches subjected to symmetrical loads. The head of the machine was fitted with a special casting H1 that bore upon the casting P2, bolted to the two 24-in. I beams. The load was delivered from the I beams through the 5-in. pin to the lever L1, thence through the lever L2 and the hangers HR1, HR2, and HR3 to the transverse levers L3 that extended across the top of the arch. The load was transmitted from H1 to P2 through a 3-in. cold rolled steel round bearing upon the plane top of P2; the load was delivered to the arch from the transverse lever L3 through a ¾-in. steel ball, inserted in the lever L3 and resting in a slight depression in a tool-steel plate bearing upon the top of the loading shelf of the arch. This detail is illustrated in Fig. 5. All other contacts in the lever-and-hanger system were tool-steel knife-edges resting on the plane surfaces of tool-steel inserts. In addition to the knife-edges in the levers L1 and L2 normal to the plane of the rib, there was also a horizontal knife-edge at the top and bottom of each hanger parallel to the plane of the rib, as illustrated in Fig. 5. These knife-edges prevented the hangers from exerting any horizontal force or lateral resistance upon the arch. They were provided especially for the tests to determine the effect of the slenderness of an arch upon its strength, but they were used in all of the tests.

The bearing of H1 upon P2 is midway between the 5-in. pins in the I beams, making the load on the arch symmetrical about its center. The levers L1 and L2 are so proportioned that, with fixed abutments and symmetrical loading, the thrust line remains within the kern of the rib over its entire length. In order that the position of the vertical reaction of the abutments might be determined, the concrete abutments were carried upon the steel castings, L4 and L4, each of which has two supports, one a knife-edge bearing upon the steel casting L5

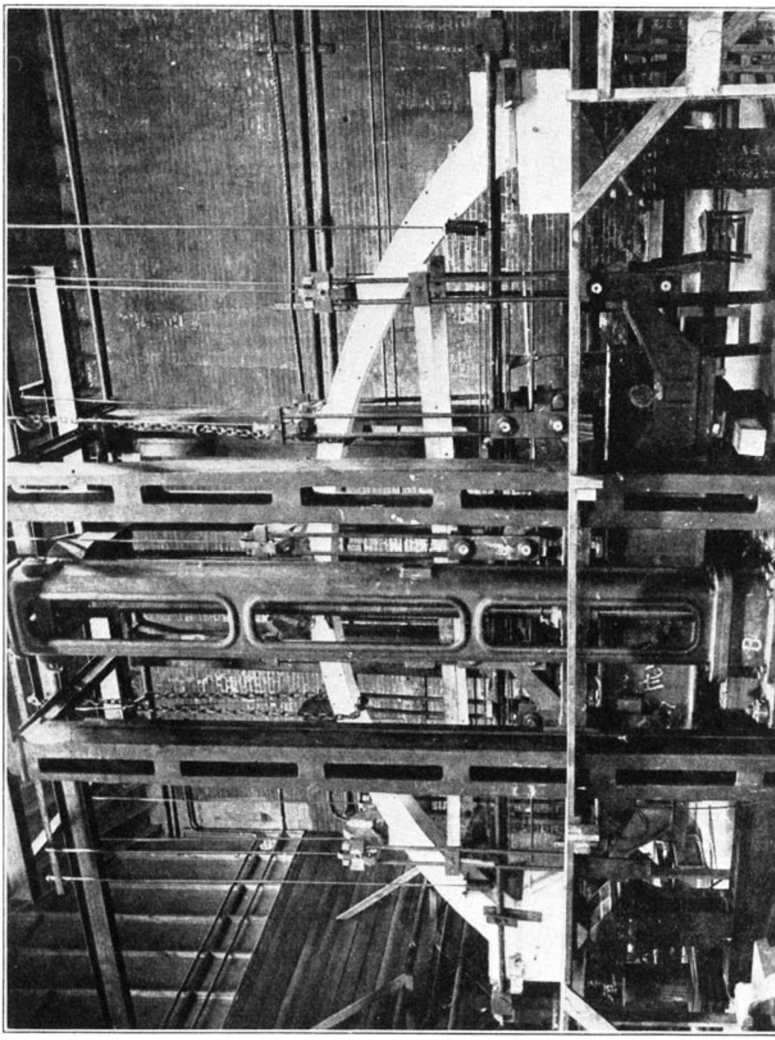


FIG. 3. ARCH IN TESTING MACHINE

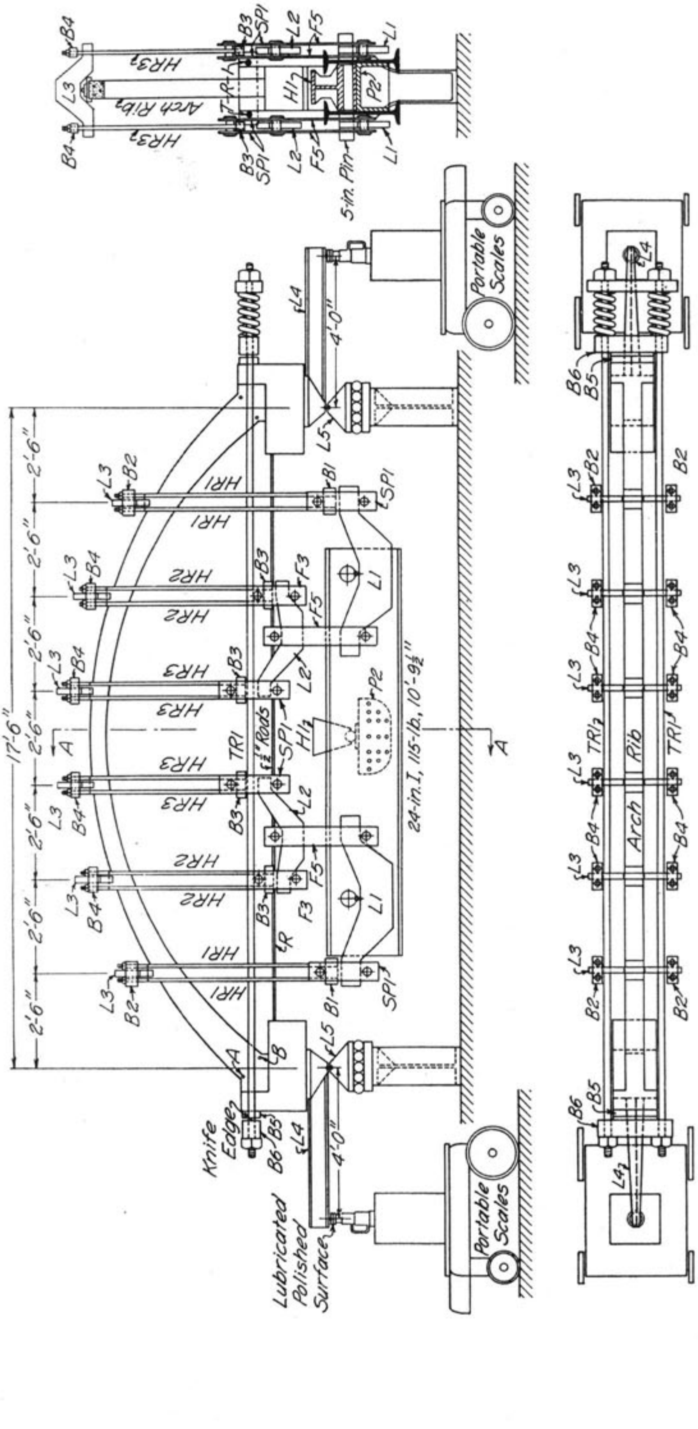


FIG. 4. LOADING MECHANISM FOR PRODUCING A SYMMETRICAL LOAD UPON AN ARCH

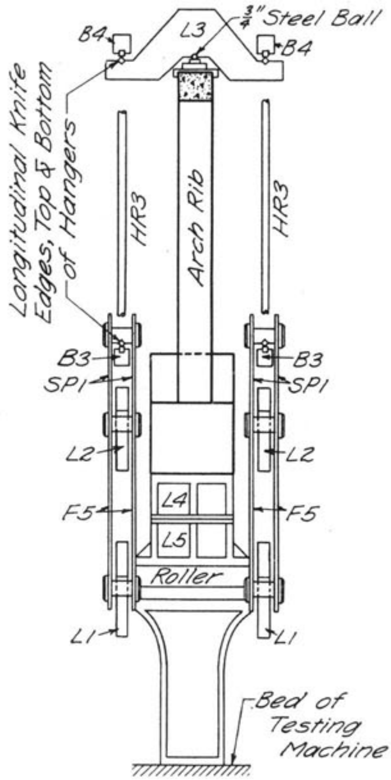


FIG. 5. DETAILS OF LOADING APPARATUS

and the other a $\frac{3}{4}$-in. steel ball bearing upon a jack. The load going through L5 was weighed by the testing machine and that going through the jack was weighed by the portable scales. The two components of the vertical reaction being thus known in amount and position, their resultant could be determined.

The casting L5, which carried most of the load, was mounted on rollers and the jack, which carried only a small portion of the load, had a special cap consisting of two discs with polished and lubricated surfaces in contact. The object of the rollers and the lubricated surfaces was to prevent the abutment supports from exerting any horizontal force upon the abutments, thereby causing all of the horizontal thrust from the arch to be transmitted to the tie rods TR1.

The angular position of an abutment was controlled by manipulating the jack under the outer end of the lever L4. The abutments usually rotated slightly as increments of the load were added and, in

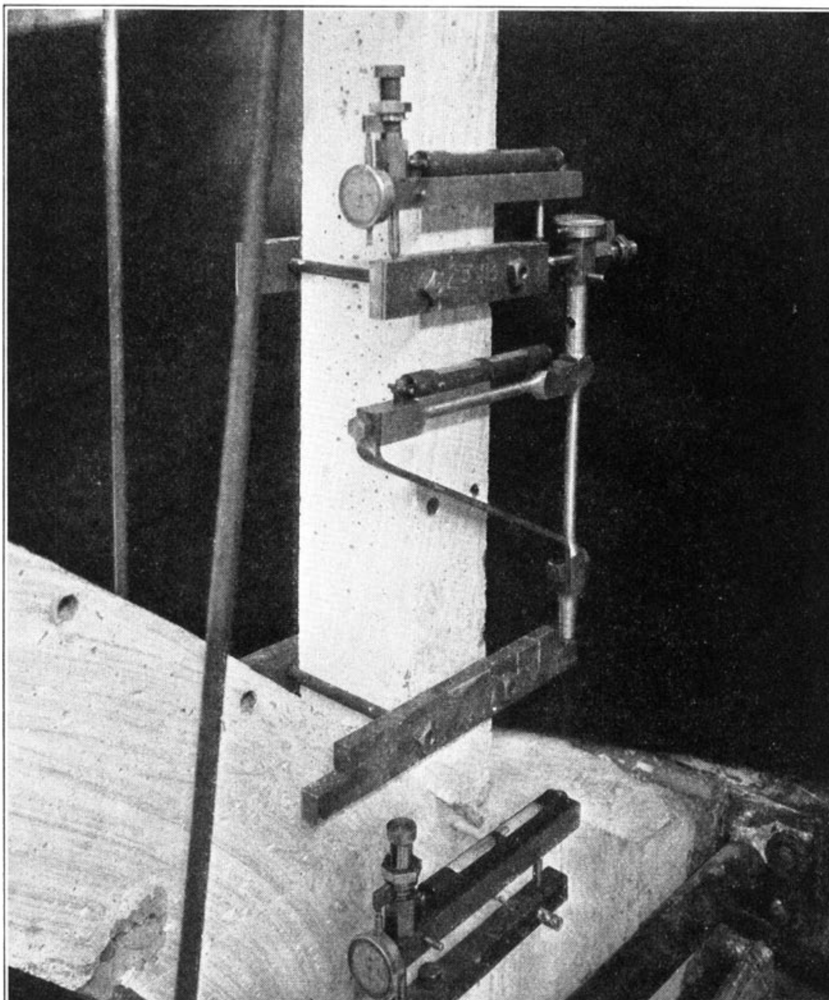


Fig. 6. Portable Level Bar for Determining Angular Position of Abutments

tests with fixed abutments, it became necessary to manipulate the jacks so as to return the abutments to their original angular position. In the case of tests involving the rotation of an abutment a known amount, the jacks were manipulated until the desired rotation had been obtained.

The transverse rods A and B, extending through the rib and located on the radial section at the springing, were provided for attaching instruments to determine or control the angular position of the abutment. Two types of instruments were used. The portable level bar,* shown at the bottom of Fig. 6, was used in the early tests. It can be used to determine the magnitude of a rotation or to adjust the abutment to its initial angular position. In later tests, the portable level bar was replaced by a level bubble attached directly to the horizontal top member of the triangular frame upon which the portable level bar at the bottom of Fig. 6 rests. The attached bubble proved more convenient in tests for which the angular position of the abutments remained constant, but it could not be used to measure the magnitude of a rotation. When the angular movement of the abutments was measured by the level bar, readings were taken on both the east and west sides of each abutment, the arch standing in a north-and-south plane while being tested. When the angular position of the abutments was controlled by the attached bubble, a bubble was placed on one side only of each abutment since, if there was any twist, only one side could be returned to its initial angular position. Readings were taken with a portable level bar on the side not having an attached bubble to determine the magnitude of any possible twist. Practically no twist occurred.

The level bar used in these tests was 8 in. long, center to center of legs. This instrument is both reliable and sensitive having, as used on these tests, a tolerance of about 0.00003 radian.

The length of the span was controlled by means of the nuts on the rods TR1. Changes in the span were measured with the ½-in. rods R extending from abutment to abutment. These rods were placed in the forms before the concrete was poured and, being anchored in the abutments, they took the horizontal thrust due to dead load when the arch was being moved. After the arch had been grouted in place in the testing machine and the nuts on the tie rods TR1 had been adjusted so as to transfer the thrust from the rods R to the rods TR1, the former were cut near one abutment. In the early tests the distance between the two adjacent ends of the rods was measured with standard thickness gages. For the later tests, a short piece was removed from each rod and Ames dials were attached in the manner shown in Fig. 7. The portion of the rod R to the left of the mounting for the dial is rigidly attached to the mounting, that portion to the right slides through a hole in the mounting and operates the plunger on the dial. The dial is graduated to 0.001 of an inch and fractions of divisions were estimated. The rods R were supported at frequent intervals by means of hangers carried by the tie rods TR1.

*This instrument is described in detail in "The Effect of Climatic Changes Upon a Multiple-Span Reinforced Concrete Arch Bridge," Univ. of Ill. Eng. Exp. Sta. Bul. 174.

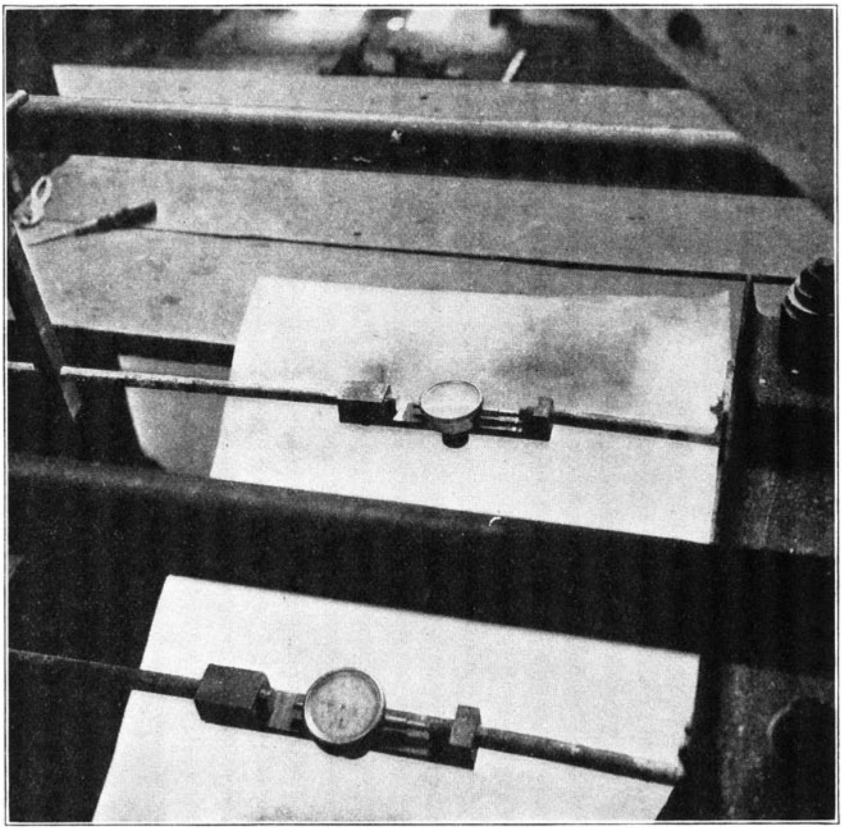


FIG 7. AMES DIALS FOR MEASURING CHANGES IN SPAN

The method of controlling the length of the span was as follows: Both dial readings were recorded before any load was applied to the arch. If the abutments were to remain fixed during a test, the load was added in small increments and the dial readings were brought to approximately their initial value by manipulating the nuts on TR1 after each increment. When the load had been brought to a predetermined value the nuts were carefully adjusted until the dial readings were brought very close to their initial value, the jacks being manipulated at the same time so as to give the abutments their correct angular position. If, instead of operating with fixed abutments, the span was to be altered, the nuts were turned until the desired change in span had been obtained.

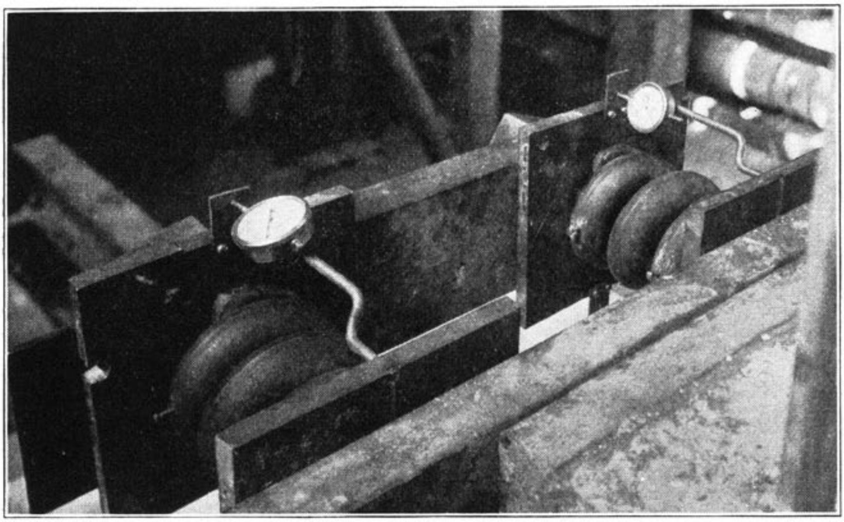


FIG. 8. SPRINGS FOR MEASURING SMALL THRUST IN TIE RODS

The apparatus was very sensitive, a change in one variable, such as the angular position of an abutment, changing the other readings, and it was only by exercising patience that all of the variables could be brought to the desired value at the same time. Readings of the variables were recorded so that if exact values were not obtained, slight differences could be noted, and the proper corrections, which generally were quite small, could be made.

The horizontal thrust was delivered to the tie rods through the steel blocks B5 and B6. The block B5 was grouted to the end of the abutment, and the contact between B5 and B6 was a horizontal knife-edge normal to the plane of the arch.

In the early tests the stress in the tie rods was measured by means of extensometers having a gage length of 12 feet, attached one to each rod. Each extensometer was calibrated by attaching it to the rod with which it was to be used, placing the rod in a testing machine, and noting the change in dial readings as the rod was subjected to various loads. The two extensometers were found to have the same constant, an increment in the load on one tie rod of 320 pounds causing a change in the dial reading of one division. That is, in testing an arch, a change of one division in each dial reading indicated that the total thrust had changed by 640 pounds.

The attached extensometers were reasonably sensitive when the thrust was large, but they were not sensitive enough to be used in

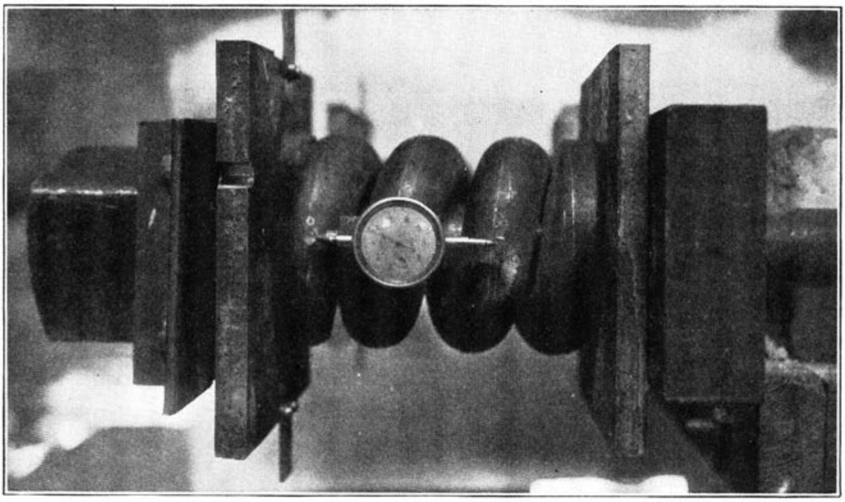


FIG. 9. SPRINGS FOR MEASURING LARGE THRUST IN TIE RODS

studying the effect of abutment movements when the thrust increments were small and, under the latter conditions, calibrated springs were used instead. Different schemes were used for measuring the thrust by means of springs, but the method shown in Fig. 8 was finally adopted. With this arrangement the compression of each spring was measured by two Ames dials, one above the spring and shown in the figure, and the other below the spring and not shown. The plates against which the springs pressed were mounted on knife-edges and the ends of the springs were faced in a lathe. The parts of the apparatus were marked so that they could be assembled in the same manner when the springs were being calibrated as when they were being used in a test. The springs used to measure the variations in the thrust when the abutments were being moved had a stiffness such that an increase of about 25 pounds in the thrust on the arch would produce a change of one division in the readings of all dials. For some of the tests three springs were used instead of two, and for the early tests the plates pressing against the springs were not mounted on knife-edges. In all cases, however, the springs were mounted in the same manner when calibrated as when used in a test.

Springs were used to measure the thrust in the tie rods throughout the test for the later arches, light springs being used to measure the effect of abutment movements, and heavy ones to measure the effect of loads. The deflection of the heavy springs was measured as shown in Fig. 9.

Steel pins were driven firmly into holes drilled in the coils of the spring in such a manner that an Ames dial with conical ends on the plunger and mounting could be placed with the conical ends in holes in the pins, thus giving the distance between the pins. There was a pair of pins on the near and the far side of each spring and the one dial with mounting was used to take all readings. This method of reading spring deflections is not as convenient as the one shown in Fig. 8, but it has the advantage that the deflection of the spring supports does not affect the dial readings. The Ames dial with its special mounting was checked against a standard bar to detect accidental changes in the instrument.

The stiffness of the springs used in measuring the thrust when the arch was loaded to destruction was such that it required an increase in the thrust of approximately 240 pounds to make a change of one division in a dial reading.

A spring is not an extremely accurate instrument for measuring forces, but the springs seemed more reliable and somewhat more sensitive than the attached extensometer that was used in the early tests. It is believed that a spring having deflections measured in the manner illustrated in Fig. 9 will give forces accurately within 2 or 3 per cent.

The strain in the concrete and in the steel was measured on two gage lines on the extrados and two on the intrados, on a section midway between each pair of adjacent load points, seven sections in all. The location of these sections is shown in Fig. 10. The gage holes for measuring the strain in the concrete were in the ends of steel rods extending radially through the rib as shown in Fig. 2. The reinforcing steel was accessible for strain readings because of the holes in the concrete formed by stools made of steel conduit set in the manner illustrated in Fig. 2. The steel stool, remaining in place during the tests, compensated for the loss of concrete due to the hole. The gage length was eight inches for both the steel and the concrete. A special strain gage having a long slender leg was used to read the strain in the steel. It had a nominal multiplication ratio of 5 to 1.5; the instrument used to read the strain in the concrete had a nominal multiplication ratio of 5 to 1.0. Both instruments were calibrated, and the true multiplication ratio was used in determining the results of the tests.

The loading apparatus that has been described and that is illustrated in Fig. 4 was used in testing arches subjected to symmetrical loading. The loading apparatus used in testing arches 27-3 and 27-4 subjected to unsymmetrical loads is similar to the apparatus shown in Fig. 4, except that H1, instead of bearing on P2 at a point half way

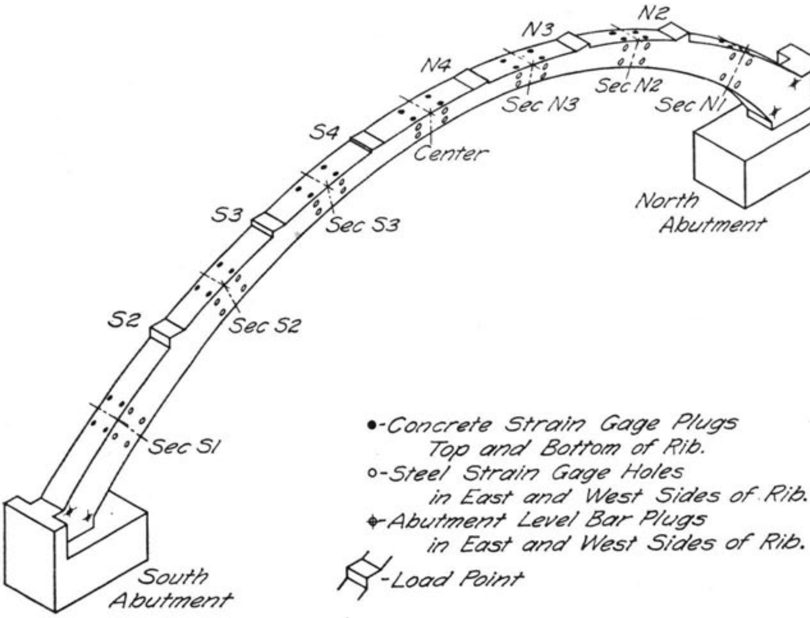


FIG. 10. SECTIONS AT WHICH STRAIN GAGE READINGS WERE TAKEN

between the two 5-in. pins, bears 10.35 inches north of the mid-point, thereby making the load on the north half of the arch 50 per cent greater than the load on the south half. This condition of loading has been made to simulate the condition of dead load over the entire span and live load over one-half the span, the load per unit length being one-half as great for live load as for dead load.

The apparatus for testing arches under unsymmetrical loading included a weighing table resting on the wing of the testing machine in place of the pedestal that supported the rollers under L5 at the right-hand end of the arch. This weighing table measured the load coming upon L5 at this end; by subtracting this load from the sum of the loads on the two castings L5, measured by the testing machine, the load on the L5 at the left-hand end of the arch could be determined. During the tests in which the arch was subjected to an unsymmetrical load variations in the relative height of the abutments, due to vertical movements of the wings of the testing machine, were measured with a wye level.

The apparatus that has been described, except the standard instruments such as Ames dials, level bars, and strain gages, was designed and built especially for these tests. The requirements were: (1) the abutment movements were to be entirely controlled, so that rotation and spread could be prevented, or so that any desired rotation or spread could be produced and measured; (2) the reactions of the abutments were to be entirely determined, so that the horizontal and vertical components of each abutment reaction could be obtained in magnitude and position; (3) the load was to be distributed to the various load points in accordance with a predetermined plan; (4) the loading apparatus was to produce no lateral force upon the arch and offer no resistance to its lateral movement; and (5) the strain in the concrete and in the steel was to be determined at a number of sections at each load. The apparatus that was used was quite complicated but it functioned well and, it is believed, met all of the requirements stipulated.

In making a test all instruments were read twice after each increment of load had been applied, the initial set of readings generally being completed before the check set was begun. Readings were generally accepted if two successive values agreed within 1.5 divisions of the dial for the strain gages, and if they agreed within one division for the level bar, except near the end of a test, when the time yield in the concrete at high stresses, occurring in the interval between the original and the check readings, caused the two sets of readings to differ by more than the established tolerances. In general the differences between two successive sets of observations were much less than the arbitrarily established tolerances given.

5. *Strength of Concrete.*—Three 6-in. by 12-in. control cylinders were poured during the construction of each arch, each cylinder being taken from a different batch of concrete. The strength of the concrete, as determined by tests of these cylinders, is given in Table 2. The values given for 26-9 and 26-10 are extremely low. Inasmuch as the mix is the same for these as for the others, and inasmuch as there was nothing about the behavior of the arches indicating that they were made of extremely poor concrete, it would seem that some mistake had been made in connection with these cylinders.

III. Properties of Arch Rib

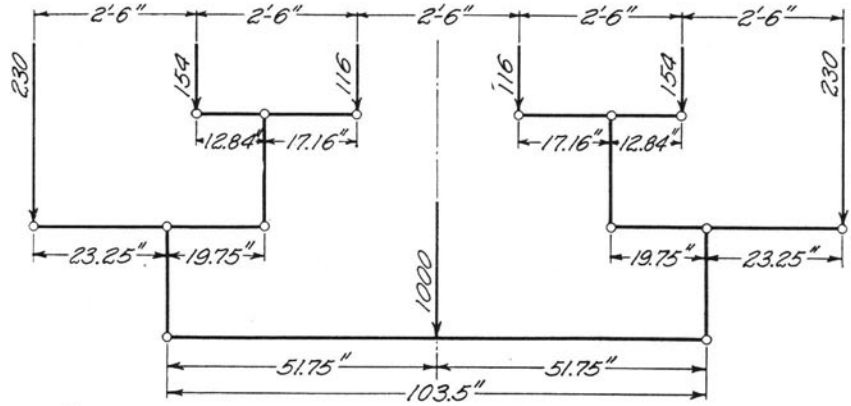
6. *Assumptions upon Which Analysis is Based.*—A number of methods have been developed for determining the reactions and stresses in a hingeless arch. These methods differ, however, only in the way in which the geometrical relations are manipulated, and are fundamentally the same. They all make use of the elastic properties of the arch to obtain three equations that, combined with the three

TABLE 2

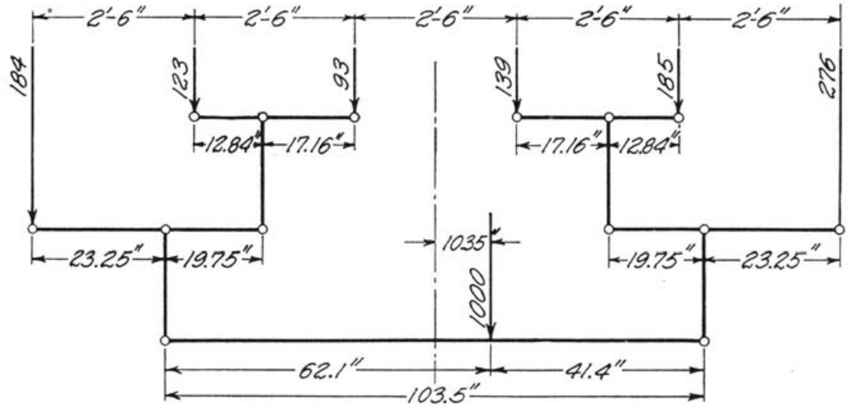
STRENGTH OF CONCRETE AS DETERMINED BY TESTS OF 6-IN. BY 12-IN. CYLINDERS

Arch No.	Cylinder No.	Age When Tested	Ultimate Strength lb. per sq. in.
26–1	1 2 3	36 36 36	2700 2390 2600 Av. 2563
26–2	1 2 3	33 33 33	2830 2660 2290 Av. 2593
26–3	1 2 3	33 33 33	2875 3490 2980 Av. 3115
26–4	1 2 3	31 31 31	3040 2355 2625 Av. 2673
26–5	1 2 3	34 34 ..	2105 2840 .... Av. 2473
26–6	1 2 3	37 37 37	3140 2670 3160 Av. 2990
26–7	1 2 3	35 35 35	3540 2480 3100 Av. 3040
26–8	1 2 3	35 35 35	3190 2220 2260 Av. 2557
26–9	1 2	30 30	1512 1510 Av. 1511
26–10	1 2 3	30 30 30	1190 1630 1370 Av. 1396
27–3	1 2 3	55 55 55	3443 3447 3250 Av. 3382
27–4	1 2 3	54 54 54	3580 3371 3785 Av. 3580

fundamental equations of static equilibrium, enable us to determine the six components into which the two abutment reactions may be resolved. If the same assumptions are used in the application of these methods they will all give the same values, except that the error resulting from the substitution of a summation of small finite quantities for the integration of differential quantities, is slightly greater for some methods than for others.



(a)-Distribution for 1000-lb. Symmetrical Load



(b)-Distribution for 1000-lb. Unsymmetrical Load

FIG. 11. LEVER SYSTEM AND LOAD DISTRIBUTION FOR ARCHES SUBJECTED TO (a) SYMMETRICAL LOAD AND (b) UNSYMMETRICAL LOAD

Any analysis of an arch is based upon assumptions as to the effect of stress upon the geometrical properties of the elements of the structure. The assumptions in the case of the experimental arch were:

(1) that stress is proportional to strain;

(2) that plane sections before deformation remain plane after deformation;

(3) that concrete resists tension, and that the modulus of elasticity is the same in tension as in compression;

(4) that the modulus of elasticity of the concrete is the same at all sections and at all stresses.

TABLE 3

REACTIONS AT NORTH ABUTMENT DUE TO UNIT LOADS AND UNIT MOVEMENTS OF ABUTMENTS

Load Point		Abutment Reactions*		
		Moment	*H*	*V*
1-lb. Load at	N2	−17.802	0.197	0.966
	N3	−12.563	0.681	0.845
	N4	+5.571	1.121	0.630
	S4	+17.915	1.121	0.370
	S3	+14.920	0.681	0.155
	S2	+4.990	0.197	0.034
Rib Shortening Effect $H = 1$ lb.		−1.138	0.0287	0.000
Movement of North Abutment	Spread† 0.10 in.	−63 543	1603	0.000
	Settlement 0.10 in.	+9039	0000	86
	Rotation‡ 0.001 Radian, Top Tipping In	+37 279	654	90
Rotation of South Abutment.‡ 0.001 Radian, Top Tipping In		+18 297	654	90

A plus sign (+) indicates that the flexural stress on the bottom of the rib is tension.
*Effect of Rib shortening not included for Loads.
†Effect of Rib shortening included for Spread.
‡Effect of Rib shortening not included.
Forces are in pounds and moments are in inch-pounds.
$E = 2\ 500\ 000$ lb. per sq. in.
Values are for rib 6⅛ in. wide, reinforced with four ½-in. round plain bars.

These assumptions are implied, even though not always stated, in the analyses usually accompanying the design of an arch. All of them are probably erroneous, and some of them are known to be considerably in error. The extent to which the errors in the assumptions affect the accuracy of the analyses based upon them is not generally known, and constituted one of the questions to be studied experimentally in this investigation.

7. *Relation between Loads, Abutment Movements, and Reactions.*—The proportions of the levers, and the distribution of a total load of 1000 pounds among the load points, are given in Fig. 11a for an arch subjected to a symmetrical load, and in Fig. 11b for one subjected to an unsymmetrical load. The reactions on the north abutment due to unit loads at the various load-points, and to unit movements of the abutments, spread, settlement, and rotation, are given in Table 3. The abutment reactions, based upon the unit values in Table 3, are given

TABLE 4

THEORETICAL ABUTMENT REACTIONS DUE TO A 1000-LB. SYMMETRICAL LOAD

Load Point	Load	Abutment Reactions		
		H	*V*	*M*
N2	230	45.3	222.1	−4095
N3	154	104.9	130.2	−1935
N4	116	130.0	73.1	+646
S4	116	130.0	42.9	+2078
S3	154	104.9	23.8	+2298
S2	230	45.3	7.9	+1148
Total, Rib Shortening Not Included	1000	560.4	500.0	+140
Effect of Rib Shortening		−18.0	00.0	−713
Total, Rib Shortening Included		542.4	500.0	−573

Forces are in pounds, moments in inch-pounds.
A plus sign (+) indicates that the flexural stress in the bottom of the rib is tension.
E = 2 500 000 lb. per sq. in.
Values are for rib 6⅛ in. wide, reinforced with four ½-in. plain round bars.

in Table 4 for an arch carrying a symmetrical load of 1000 pounds, and in Table 5 for an arch carrying an unsymmetrical load of the same magnitude.

The tangential thrust, moment, and unit stress in the concrete at various sections of the rib, as determined by the elastic theory, are given in Table 6 for a 1000-lb. symmetrical load and in Table 7 for a 1000-lb. unsymmetrical load. The geometrical properties of the arch ribs are given in Table 8.

IV. DESCRIPTION OF TESTS

8. *Description of Tests.*—This bulletin contains the report of tests of thirteen arches. Specimens 26-1 to 26-11, inclusive, were subjected to symmetrical loads, and 27-3 and 27-4 to unsymmetrical loads. The initial load for each arch consisted of the weight of the arch and the apparatus. This load was distributed in a manner slightly different from that in which the load derived from the testing machine was distributed and, since it was small compared with the total load, it was taken as the zero or basic load in studying the relation between increments of load and increments of abutment reactions, stresses, and strains. The ultimate load reported for each arch, however, includes the weight of the arch and the apparatus. Details relative to the arches are given in Table 9, and relative to the tests in Tables 10 to 22, inclusive.

TABLE 5

THEORETICAL ABUTMENT REACTIONS DUE TO A 1000-LB. UNSYMMETRICAL LOAD

The rate of loading is 50 per cent greater on the north than on the south half of the arch

Load Point	Load	North Abutment Reactions			South Abutment Reactions		
		H	*V*	*M*	*H*	*V*	*M*
N2	276	54.3	266.5	−4913	54.3	9.5	+1377
N3	185	126.0	156.4	−2324	126.0	28.7	+2760
N4	139	155.8	87.6	+774	155.8	51.4	+2490
S4	93	104.2	34.4	+1666	104.2	58.6	+518
S3	123	83.8	19.0	+1835	83.8	103.9	−1545
S2	184	36.2	6.3	+918	36.2	177.7	−3276
Total, Rib Shortening Not Included	1000	560.3	570.2	−2044	560.3	429.8	+2324
Effect of Rib Shortening		−18.0	000.0	−713	−18.0	000.0	− 713
Total, Rib Shortening Included		542.3	570.2	−2757	542.3	429.8	+1611

Forces are in pounds, moments are in inch-pounds; a plus (+) sign indicates that the flexural stress in the bottom of the rib is tension.
E = 2 500 000 lb. per sq. in.
Values are for rib 6⅛ in. wide, reinforced with four ½-in. plain round bars.

TABLE 6

TANGENTIAL THRUST, MOMENT, AND STRESS AT VARIOUS SECTIONS DUE TO A 1000-LB. SYMMETRICAL LOAD

Determined by the elastic theory

Section	Tang. Thrust	Moment	Unit Stress in lb. per sq. in.	
			Top	Bottom
N1	737.1	−1107	− 1.5	−21.5
N2	605.8	− 163	− 9.6	−14.8
N3	554.7	− 37	−12.4	−14.2
Center	542.5	− 66	−12.0	−15.6
S3	554.7	− 37	−12.4	−14.2
S2	605.8	− 163	− 9.6	−14.8
S1	737.1	−1107	− 1.5	−21.5

For stress (+) indicates tension and (−) compression.
For moment (+) indicates that the flexural stress is tension on the bottom of the rib.
E = 2 500 000 lb. per sq. in.
The horizontal thrust is 542.5 lb.
Values are for rib 6⅛ in. wide, reinforced with four ½-in. plain round bars.

TABLE 7

TANGENTIAL THRUST, MOMENT, AND STRESS AT VARIOUS SECTIONS DUE TO A 1000-LB. UNSYMMETRICAL LOAD

Determined by the elastic theory

Section	Tang. Thrust	Moment	Unit Stress in lb. per sq. in.	
			Top	Bottom
N1	782.1	−2235	+ 7.9	−32.3
N2	616.0	+ 129	−14.5	−10.3
N3	553.0	+ 518	−25.7	− 0.9
Center	542.3	− 59	−12.2	−15.4
S3	556.0	− 574	+ 0.4	−27.0
S2	595.3	− 441	− 5.0	−19.0
S1	691.6	+ 27	−11.0	−10.6

For stress (+) indicates tension and (−) compression.
For moment (+) indicates that the flexural stress is tension on the bottom of the rib.
E = 2 500 000 lb. per sq. in.
The horizontal thrust is 542.5 lb.
Values are for rib 6⅛ in. wide, reinforced with four ½-in. plain round bars.

TABLE 8

GEOMETRICAL PROPERTIES OF ARCH RIBS

Section	X Referred to North Springing	Y Referred to North Springing	Depth of Rib	Angle Tangent to Axis Makes with Horiz.	Equiv. Area of Section*	Area of Concrete	Moment of Inertia*
Arch 26-2†							
N1	15	14.81	9.06	40° 05′	62.11	54.89	474
N2	45	34.36	6.67	25° 20′	47.47	40.25	195
N3	75	44.80	5.40	12° 40′	39.70	32.47	106
Center	105	48.06	5.00	0° 0′	37.25	30.02	85
Arches 26-5 to 26-11, 27-3, and 27-4							
N1	15	14.81	9.06	40° 05′	64.13	54.71	503
N2	45	34.36	6.67	25° 20′	49.49	40.07	209
N3	75	44.80	5.40	12° 40′	41.72	32.29	113
Center	105	48.06	5.00	0° 0′	39.27	29.84	91

*n = 12. The moment of inertia is based upon the assumption that the concrete is stressed over the entire section.

†The equivalent area of section, area of concrete, and moment of inertia are the same per inch width of rib for arches 26-1, 26-3, and 26-4 as they are for 26-2.

TABLE 9

DETAILS OF ARCHES

No. of Specimen	Width of Rib in.	Type and Size of Reinforcing		Date of Pouring	Date of Testing	Age at End of Test in Days	Kind of Loading
		Longitudinal	Spiral				
26-1	8	4 ½-in. plain round steel rods	None	Jan. 5, 1926	Feb. 5-9, 1926	35	Symmetrical
26-2	6⅛	4 7/16-in. plain round steel rods	None	Jan. 21, 1926	Feb. 18-20, 1926	30	Symmetrical
26-3	4½	4 ⅜-in. plain round steel rods	None	Feb. 1, 1926	Mar. 1-2, 1926	29	Symmetrical
26-4	3⅛	4 5/16-in. plain round steel rods	None	Feb. 12, 1926	Mar. 13, 1926	29	Symmetrical
26-5	6⅛	4 ½-in. plain round steel rods	None	Feb. 2, 1926	Mar. 25-27, 1926	31	Symmetrical
26-6	6⅛	4 ½-in. plain round steel rods	¼-in. round rods 1-in. pitch 5-in. diameter	Mar. 6, 1926	Apr. 5-8, 1926	33	Symmetrical
26-7	6⅛	4 ½-in. plain round steel rods	None	Mar. 9, 1926	Apr. 15-17, 1926	39	Symmetrical
26-8	6⅛	4 ½-in. plain round steel rods	¼-in. round rods 1-in. pitch 5-in. diameter	Apr. 1, 1926	Apr. 28 to May 1, 1926	30	Symmetrical
26-9	6⅛	4 ½-in. plain round steel rods	None		May 18-19, 1926	30 (Approx.)	Symmetrical
26-10	6⅛	4 ½-in. plain round steel rods	¼-in. round rods 1-in. pitch 5-in. diameter		June 4-5, 1926	30 (Approx.)	Symmetrical
26-11	6⅛	4 ½-in. plain round steel rods	¼-in. round rods 1-in. pitch 5-in. diameter	May 13, 1926	June 12 to Sept. 20, 1926	30 at beginning of test	Symmetrical
27-3	6⅛	4 ½-in. plain round steel rods	None	Jan. 18, 1927	Mar. 11-12, 1927	53	Unsymmetrical
27-4	6⅛	4 ½-in. plain round steel rods	None	Feb. 7, 1927	Mar. 31, 1927	51	Unsymmetrical

TABLE 10

DETAILS OF TEST, ARCH 26-1

Details of Arch and Loading	Remarks
Width: 8 inches Reinforcing: Longitudinal only. Loading: Symmetrical	Object of test: (1) To determine the effect of the slenderness ratio of a rib upon its strength; (2) to determine the stress distribution in a rib due to load; and (3) to determine the strength developed by concrete in an arch. Recorded data: (1) The position and magnitude of the vertical and horizontal components of the abutment reactions. (2) The strain in concrete and steel at top and bottom of the rib at seven sections (see Fig. 10).
Total Load on Arch	(3) The lateral deflection at the crown.
(1) 8 630 lb. (2) 34 720 lb. (3) 57 960 lb. (4) 57 765 lb. (5) 88 900 lb. (6) 119 345 lb. (7) 8 630 lb. (8) 144 780 lb. (9) 176 635 lb. (10) 193 820 lb.	Readings: Feb. 5. A complete set of readings was taken at loads (1), (2), and (3). Load (1) consisted of weight of rib and apparatus. Load (3) remained on the arch until Feb. 8. Feb. 8. A complete set of readings was taken at loads (4), (5), and (6). The arch stood unloaded overnight. Feb. 9. A complete set of readings was taken at loads (7), (8), and (9). Arch failed at load (10). The character of fracture is shown in Fig. 12. The span and the angular position of the abutments were kept as near their original values as possible during the test. The south abutment cracked somewhat but, since the points where the span and angular position of the abutments were measured were toward the rib from the cracks, the results of the test were probably not seriously affected. The failure occurred abruptly at the crown without any warning. Control cylinders: The average strength of three cylinders was 2565 lb. per sq. in. at 36 days, one day older than the arch at the time of failure.

TABLE 11

DETAILS OF TEST, ARCH 26-2

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcing: Longitudinal only. Loading: Symmetrical	The object of the test and the method of testing were the same as for Arch 26-1. Feb. 18. A complete set of readings was taken at loads (1), (2), and (3). Load (1) consisted of the weight of the rib and apparatus. The arch stood unloaded until Feb. 20.
Total Load on Arch	
(1) 8 370 lb. (2) 29 100 lb. (3) 49 000 lb. (4) 8 370 lb. (5) 70 965 lb. (6) 91 913 lb. (7) 111 970 lb. (8) 128 600 lb. (9) 147 660 lb.	Feb. 20. A complete set of readings was taken at loads (4), (5), (6), (7), and (8). The arch failed at load (9). The failure occurred suddenly and without warning at load-point S2. The character of rupture is shown in Fig. 13. Control cylinders: The average strength of three cylinders was 2595 lb. per sq. in. at 33 days, three days older than the arch at the end of the test.

TABLE 12

DETAILS OF TEST, ARCH 26-3

Details of Arch and Loading	Remarks
Width: 4½ in. Reinforcing: Longitudinal only. Loading: Symmetrical	The object of the test and the method of testing were the same as for Arch 26-1. March 1. A complete set of readings was taken at loads (1), (2), and (3). Load (1) consisted of the weight of the rib and apparatus. The arch stood unloaded overnight.
Total Load on Arch	
(1) 8 140 lb. (2) 19 630 lb. (3) 33 710 lb. (4) 8 140 lb. (5) 52 490 lb. (6) 62 555 lb. (7) 75 365 lb. (8) 84 000 lb. (9) 84 800 lb.	March 2. A complete set of readings was taken at loads (4), (5), (6), and (7). The concrete began to spall at the crown at load (8) and failure occurred at load (9). The character of the fracture is shown in Fig. 14. Control cylinders: The average strength of three control cylinders was 3115 lb. per sq. in. at 33 days, four days older than the arch when tested.

TABLE 13

DETAILS OF TEST, ARCH 26-4

Details of Arch and Loading	Remarks
Width: 3⅛ in. Reinforcing: Longitudinal only. Loading: Symmetrical	The object of the test and the method of testing were the same as for Arch 26-1. March 13. A complete set of readings was taken at loads (1), (2), (3), (4), (5), (6), and (7). Load (1) consisted of the weight of the rib and apparatus.
Total Load on Arch	
(1) 7 930 lb. (2) 18 895 lb. (3) 28 545 lb. (4) 37 480 lb. (5) 48 125 lb. (6) 57 075 lb. (7) 58 400 lb. (8) 65 700 lb.	The arch began to buckle at load (7) and lateral supports were provided to prevent the rib from failing while strain-gage readings were taken. The supports were then removed and the load was increased. Failure occurred at load (8). The arch failed by buckling and probably would have failed under load (7) if the supports had not been provided. The character of the fracture is shown in Fig. 15. Control cylinders: The average strength of three cylinders was 2675 lb. per sq. in. at 31 days, two days older than the arch when tested.

TABLE 14

DETAILS OF TEST, ARCH 26-5

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcing: Longitudinal only. Loading: Symmetrical	Object of test: (1) To determine the elastic properties of an arch rib by measuring the changes in the abutment reactions accompanying movements of the abutments; (2) to determine the stress distribution of the rib due to load; (3) to determine the strength developed by concrete.
Total Load on Arch	Recorded data: (1) Strains in concrete and steel at seven sections due to loads and abutment movements. (2) Abutment reactions due to loads and abutment movements.
(1) 25 000 lb. (2) 8 370 lb. (3) 38 650 lb. (4) 62 620 lb. (5) 95 155 lb. (6) 124 065 lb. (7) 153 470 lb.	Readings: March 26. Load (1) was maintained constant and a complete set of readings was taken when the abutments were at each of a number of positions. The report of the test is contained in Table 29, Section 12. March 27. A complete set of readings was taken at loads (2), (3), (4), (5), and (6), abutments remaining fixed. Load (2) consisted of the weight of the rib and apparatus. Failure at the crown occurred at load (7). Control cylinders: The average strength of three control cylinders was 2470 lb. per sq. in. at 34 days, the age of the arch at failure.

TABLE 15

DETAILS OF TEST, ARCH 26-6

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcing: Longitudinal and spiral Loading: Symmetrical	The object and method of testing were the same as for Arch 26-5. April 5. A complete set of readings was taken at load (1). Load (1) was held constant while the relation between abutment movements and reactions was determined.
Total Load on Arch	April 6. Same procedure was followed for load (2) as for load (1). The results are reported in Table 29, Section 12.
(1) 19 000 lb. (2) 22 500 lb. (3) 8 370 lb. (4) 42 545 lb. (5) 71 215 lb. (6) 102 555 lb. (7) 132 100 lb. (8) 159 450 lb. (9) 140 000 lb.	April 8. A complete set of readings was taken at loads (3), (4), (5), (6), and (7), the abutments remaining fixed. Load (3) consisted of the weight of the rib and apparatus. Concrete outside of the spiral spalled off at load (8), as shown in Fig. 16. After the initial failure had occurred at the crown, thus reducing the load, the head of the testing machine was run down to see whether the concrete inside the spiral was capable of carrying a higher load. Load (9) was the maximum thus obtained. The arch buckled to the east at the center and the "after failure" cracks, shown in Fig. 17, appeared. The character of the failure is shown in Fig. 17. Control cylinders: The average strength of three control cylinders was 2990 lb. per sq. in. at 37 days, four days older than the arch at the completion of the test.

TABLE 16

DETAILS OF TEST, ARCH 26-7

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcement: Longitudinal only. Loading: Symmetrical Total Load on Arch (1) 21 000 lb. (2) 41 000 lb. (3) 8 370 lb. (4) 50 000 lb. held constant	This test was similar to the preceding ones with the exception that the test to destruction was made by the application of moment to the abutments rather than by the addition of the vertical load. This was done to simulate the condition where the piers of an arch rotate instead of remaining fixed, as assumed in the design. April 16. Load (1) was held constant while the relation between abutment movements and reactions was determined. April 17. The arch was subjected to the same test under load (2) as under load (1). The results of these tests are contained in Table 29, Section 12. April 19. A complete set of readings was taken at load (3), the weight of rib and apparatus, with the abutments in their normal position. A complete set of readings was taken at load (4) with abutments in their normal position. The abutments were then allowed to tip outward at the top and complete readings were taken at four angular positions. The arch did not fail, and as further rotation could not be obtained, the abutments were rotated in the opposite direction as far as possible but still failure did not occur. The arch was reset in such a manner that additional rotations could be produced, and the abutments were tipped inward at the top, complete readings being obtained at four angular positions. The arch failed by compression of the concrete on the outside of the rib at the north springing as shown in Fig. 18. The relation between the rotation and the moment at the springing is given in Fig. 52. The moments on the two abutments were maintained approximately equal throughout the test. Control cylinders: The average strength of three cylinders was 3040 lb. per sq. in. at 35 days, four days less than the age of the arch at the end of the test.

TABLE 17

DETAILS OF TEST, ARCH 26-8

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcement: Longitudinal and spiral Loading: Symmetrical Total Load on Arch (1) 20 000 lb. (2) 40 000 lb. (3) 50 000 lb. held constant	The test was similar to the test of arch 26-7. April 28-29. Load (1) was held constant while the relation between abutment movements and reactions was determined. April 30. The arch was subjected to the same test under load (2) as under load (1). The results are contained in Table 29, Section 12. The arch was tested to destruction by rotating the abutments inward at the top. Seven complete readings were taken. The relation between moment at the springing and rotation of the abutment is given in Fig. 52. The failure, which occurred by compression in the concrete at the outside of the rib at the north springing, is shown in Fig. 19. The moments on the two abutments were maintained approximately equal throughout the test. Control cylinders: The average strength of 3 cylinders was 2560 lb. per sq. in. at 35 days, five days older than the arch at the end of the test.

TABLE 18

DETAILS OF TEST, ARCH 26-9

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcement: Longitudinal only. Loading: Symmetrical	The test was similar to the test of arch 26-8. May 18. A complete set of readings was taken at load (1), weight of rib and apparatus, with the abutments in their normal position.
Total Load on Arch (1) 8 370 lb. (2) 50 000 lb. held constant	May 19. A full set of readings was taken at load (2) with the abutments in their normal position. Then, with the load and span length held constant, the moment at the abutments was increased, complete sets of readings being made after each of five increments of moment had been added. The failure, which occurred by compression in the concrete at the top of the rib at the south springing, is shown in Fig. 20. The relation between the moment and the rotation of the abutments is shown in Fig. 52. Control cylinders: The average strength of three cylinders was 1511 lb. per sq. in.

TABLE 19

DETAILS OF TEST, ARCH 26-10

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcing: Longitudinal and spiral Loading: Symmetrical	Object of test: To determine the effect of spiral reinforcement upon the strength of a rib. Method of testing: The arch was loaded to destruction under a symmetrical load, the abutments being held as nearly stationary as possible.
Total load on Arch (1) 8 370 lb. (2) 32 385 lb. (3) 64 528 lb. (4) 98 148 lb. (5) 128 520 lb. (6) 139 750 lb. (7) 147 860 lb.	June 4. A complete set of readings was taken at load (1), weight of rib and apparatus. June 5. A complete set of readings was taken at loads (2), (3), (4), and (5). Abutment readings only were taken at load (6). At load (7) the arch failed near the center by spalling of the concrete outside of the spiral reinforcing. Control cylinders: The average strength of three cylinders reported was 1396 lb. per sq. in. This value seems to be in error.

TABLE 20

DETAILS OF TEST, ARCH 26–11

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcing: Longitudinal and spiral Loading: Symmetrical Total Load on Arch (1) 8 370 lb. (2) 50 000 lb.	Object of test: (1) To determine the influence of time yield in concrete upon movements of the abutments. June 12. A complete set of readings was taken with abutments in their normal position at loads (1) and (2). Load (1) consisted of the weight of the rib and apparatus. The jack under the outer end of each abutment lever, L4, Fig. 4, was extended until the load on each scale was 3500 lb. The load applied by the testing machine was reduced in order to keep the total load equal to 50 000 lb. A complete set of readings was taken after making these adjustments. The arch remained under the influence of this load until July 2. In the meantime the load on each jack and on the testing machine was adjusted each morning except Sunday so that the total load and the moments at each springing remained constant for the 20-day period. July 2. A complete set of readings was taken. The load on each jack was then increased to 4500 lb., the total load on the arch being kept at 50-000 lb., and another complete set of readings was taken. The loads on the scales and on the testing machine were adjusted each morning from July 2 to 23, and a complete set of readings was taken on the latter date. The loads were adjusted each morning during the remaining portion of July, but adjustments were not made in August. On August 18 the loads on the north and south scales were 4280 lb. and 4230 lb., respectively, and the total load on the arch was 49 010 lb. These loads were increased to 4500, 4500, and 50 000 lb., respectively, and a complete set of readings was taken. The loads were not further readjusted in August. September 3. The loads had fallen off to 4300, 4270, and 48 320 lb., respectively. Beginning at this date they were adjusted each morning to 4500, 4500, and 50 000 lb. September 18. Another complete set of readings was taken. September 20. The arch was tested to destruction, the load being increased and the abutments held as nearly stationary as possible. Spalling of the whitewash occurred on the bottom of the rib at the crown, indicating that failure was impending, at a load of 4935 lb. on the north scale, 5106 lb. on the south scale, and a total load of 129 981 lb. on the arch. Just before failure occurred the loads were 207, 4003, and 159 245 lb., respectively. The relation between the moment on the abutments, rotation of abutments, and time are indicated in Figs. 53 and 54.

TABLE 21

DETAILS OF TEST, ARCH 27-3

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcing: Longitudinal only. Loading: Unsymmetrical	Object of test: To study the stress distribution and the strength developed by the concrete in an arch when the flexural stress is high. March 11. A complete set of readings was taken at loads (1), (2), (3), and (4). Load (1) consisted of the weight of the rib and apparatus.
Total Load on Arch	Load (4) remained on the arch overnight.
(1) 8 850 lb. (2) 26 023 lb. (3) 37 120 lb. (4) 55 631 lb. (5) 74 443 lb. (6) 79 805 lb. (7) 91 711 lb. (8) 98 701 lb. (9) 111 436 lb.	March 12. A complete set of readings was taken at load (5). First tension crack appeared at bottom of rib under load point N4 at load (5). Abutment readings only were taken at loads (6), (7), (8), and (9). The abutments were kept as nearly stationary as possible during the test. Compression failure seemed impending at load (9) near load-point N4. The arch failed at this section after standing for 15 minutes without further increase of load. The character of fracture is shown in Fig. 21. Control cylinders: The average strength of three cylinders was 3382 lb. per sq. in. at 55 days, two days older than the arch at completion of test.

TABLE 22

DETAILS OF TEST, ARCH 27-4

Details of Arch and Loading	Remarks
Width: 6⅛ in. Reinforcing: Longitudinal only. Loading: Unsymmetrical	The object of this test was the same as for Arch 27-3. Load (1) consisted of the weight of the rib and apparatus. March 31. A complete set of readings was taken at loads (1), (2), and (3). Load (3) remained on the arch overnight.
Total Load on Arch	April 1. A complete set of readings was taken at loads (4), (5), (6), and (7).
(1) 8 850 lb. (2) 25 869 lb. (3) 39 187 lb. (4) 55 123 lb. (5) 69 902 lb. (6) 82 102 lb. (7) 95 774 lb. (8) 104 117 lb. (9) 113 172 lb. (10) 135 917 lb.	First tension cracks occurred under load (6) at top of section N1. At load (7) a vertical crack formed in the north abutment beginning at the bottom and extending almost to the springing. The load was reduced to 20 000 lb. and a steel clamp was put on the abutment, care being taken not to disturb the rods measuring the span of the arch or the plugs carrying the level bar. It is believed that the crack did not seriously influence the remainder of the test. A crack appeared at the bottom of the rib under load-point N2, at load (8). A flexural failure started to develop at load-point N4 (see Fig. 22), but final failure was an abrupt compression failure at Section S3 (see Fig. 23) under load (10). Control cylinders: The average strength of three control cylinders was 3580 lb. per sq. in. at 54 days, three days older than the arch at completion of the test.

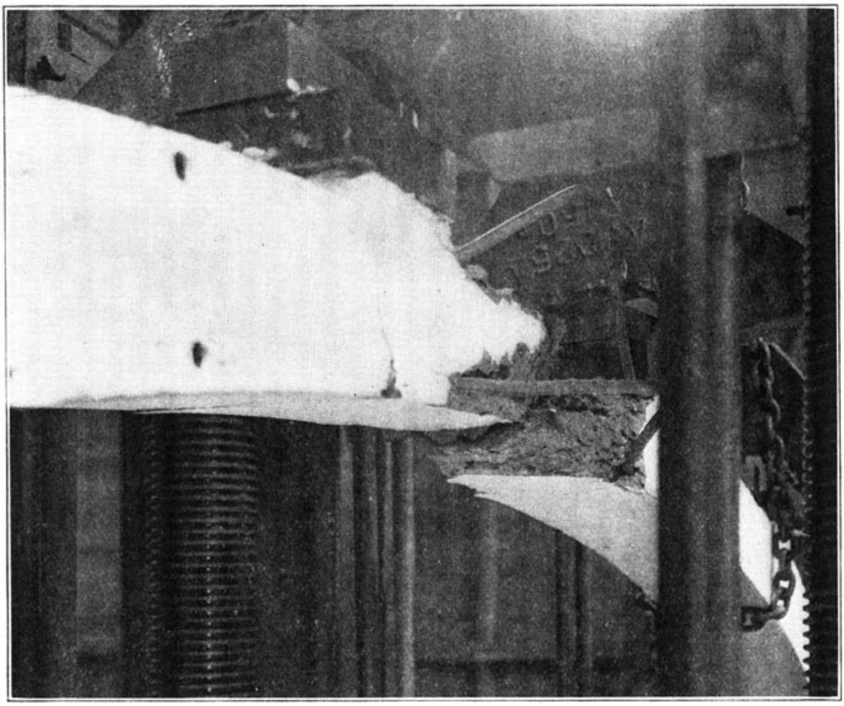


FIG. 12. FAILURE OF ARCH 26-1 AT CROWN

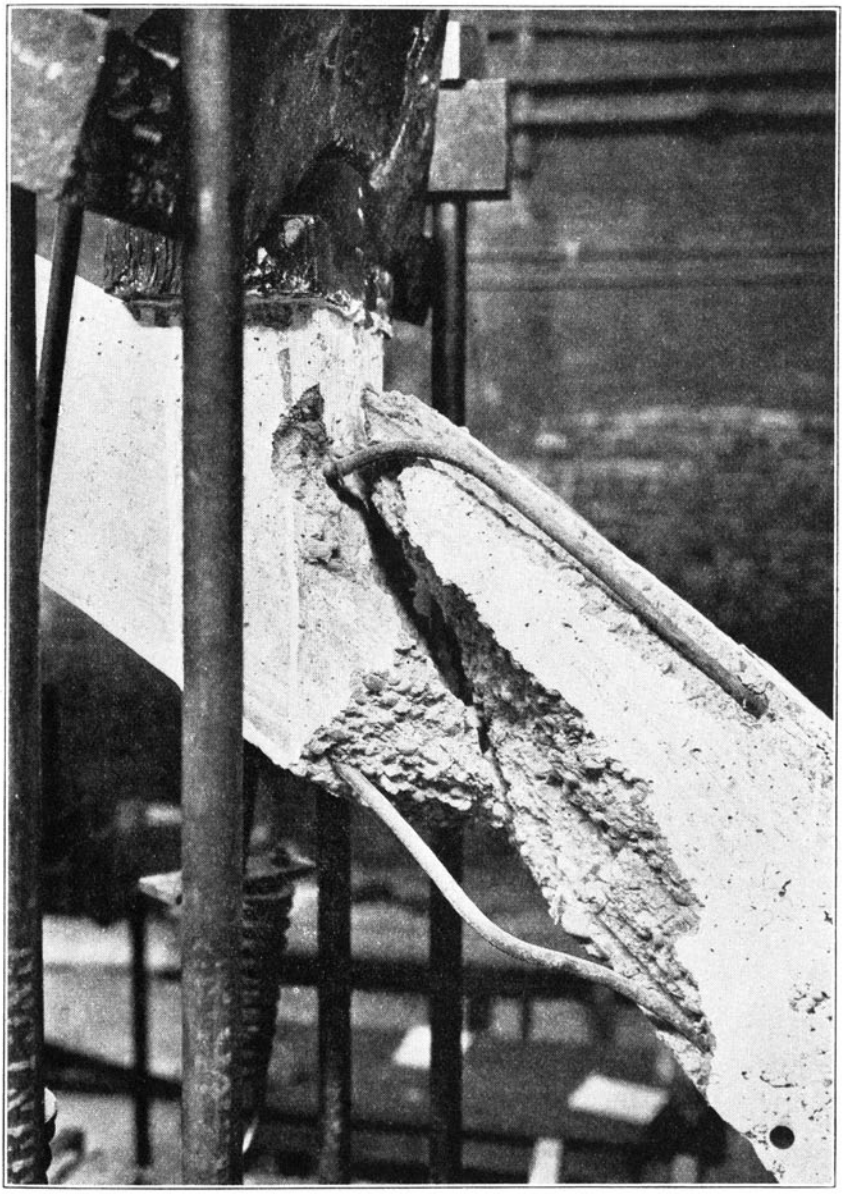


FIG. 13. FAILURE OF ARCH 26-2 AT S2

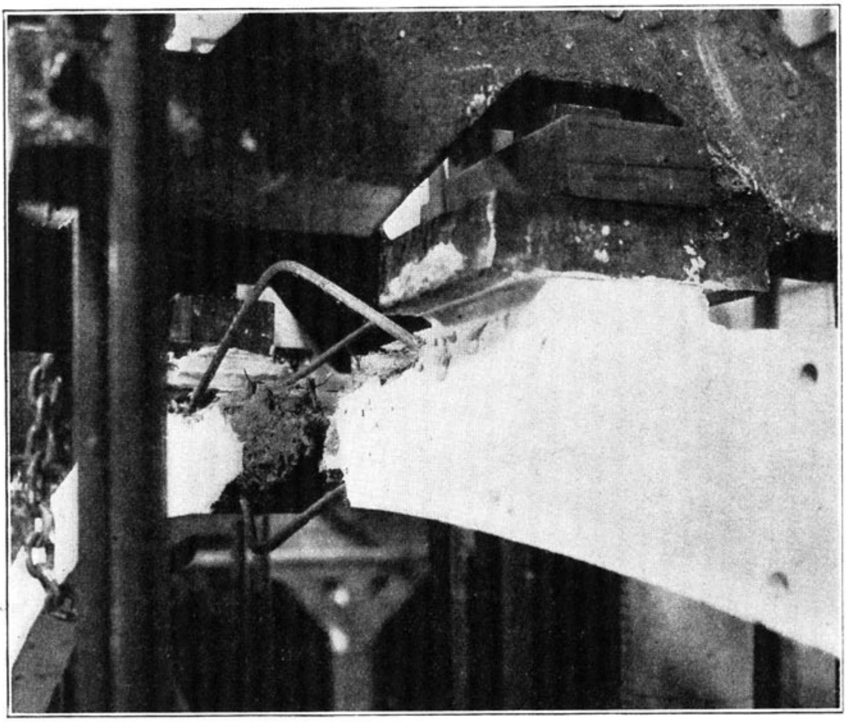


FIG. 14. FAILURE OF ARCH 26-3 AT CROWN

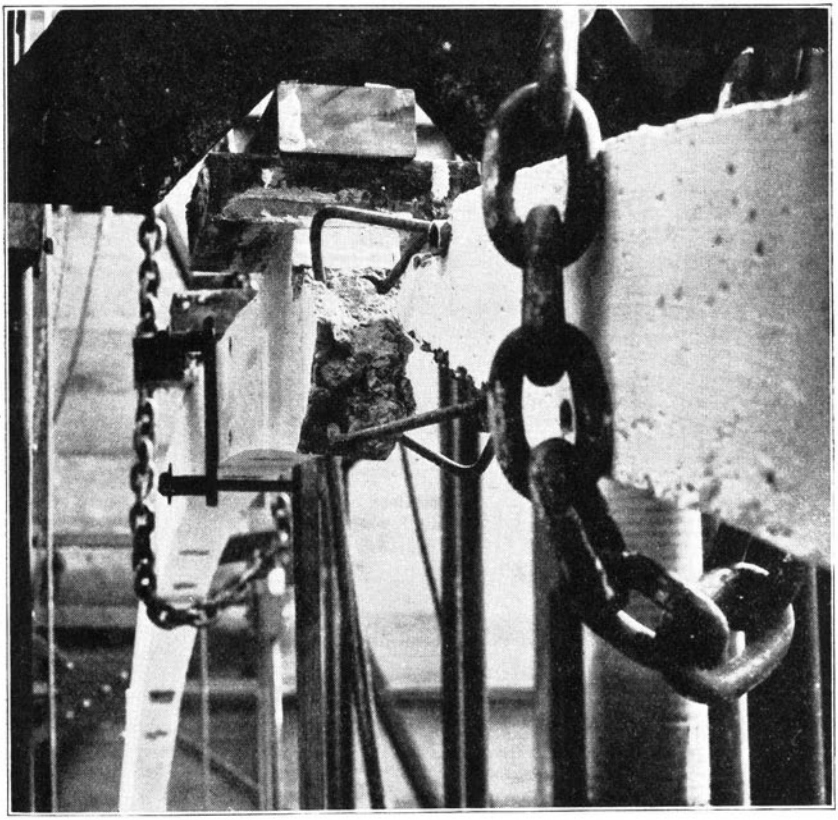


FIG. 15. BUCKLING FAILURE OF ARCH 26-4

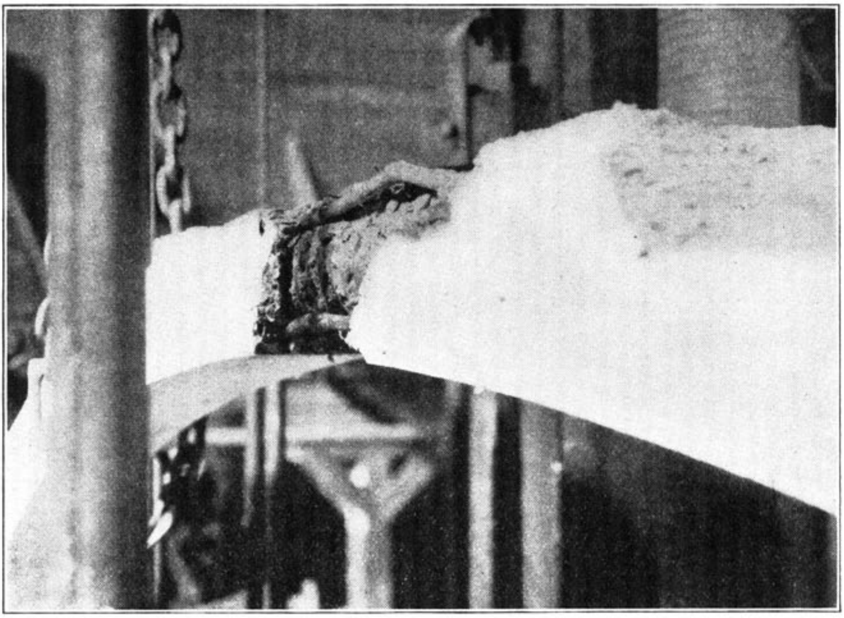
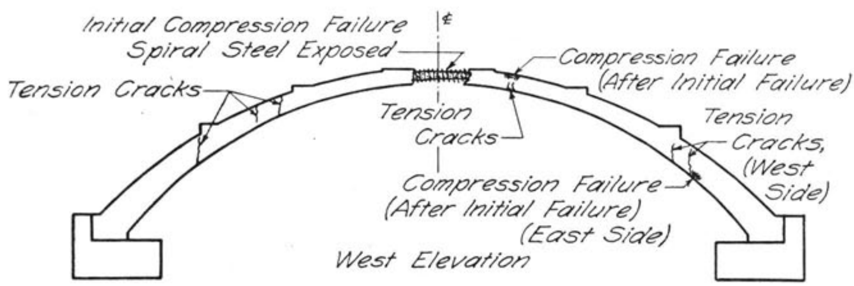


FIG. 16. FAILURE OF ARCH 26-6 AT CROWN



After the arch failed at the crown the Testing Machine was run down to see if more load would be carried. The concrete spalled away from the hooping and the maximum load, the second time, was 140 000 lb., approx. The arch buckled to the east at the center and the "after failure" cracks appeared.

FIG. 17. AFTER-FAILURE CRACKS IN ARCH 26-6

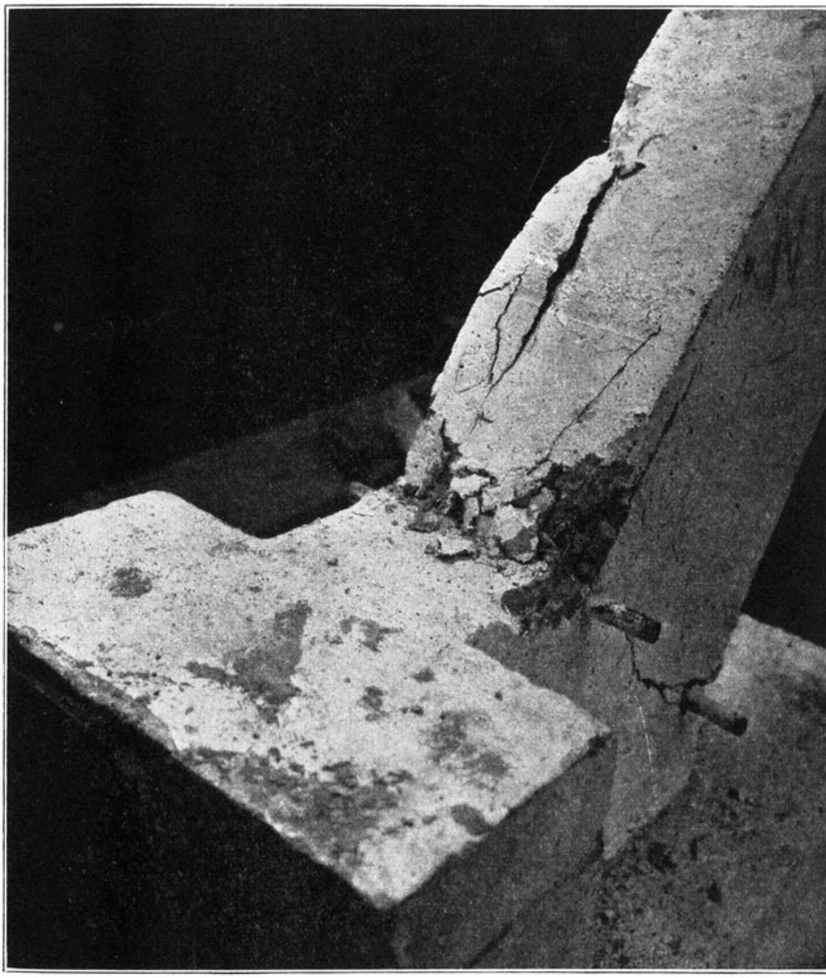


FIG. 18. FAILURE OF ARCH 26-7 AT NORTH SPRINGING DUE TO ROTATION OF ABUTMENTS

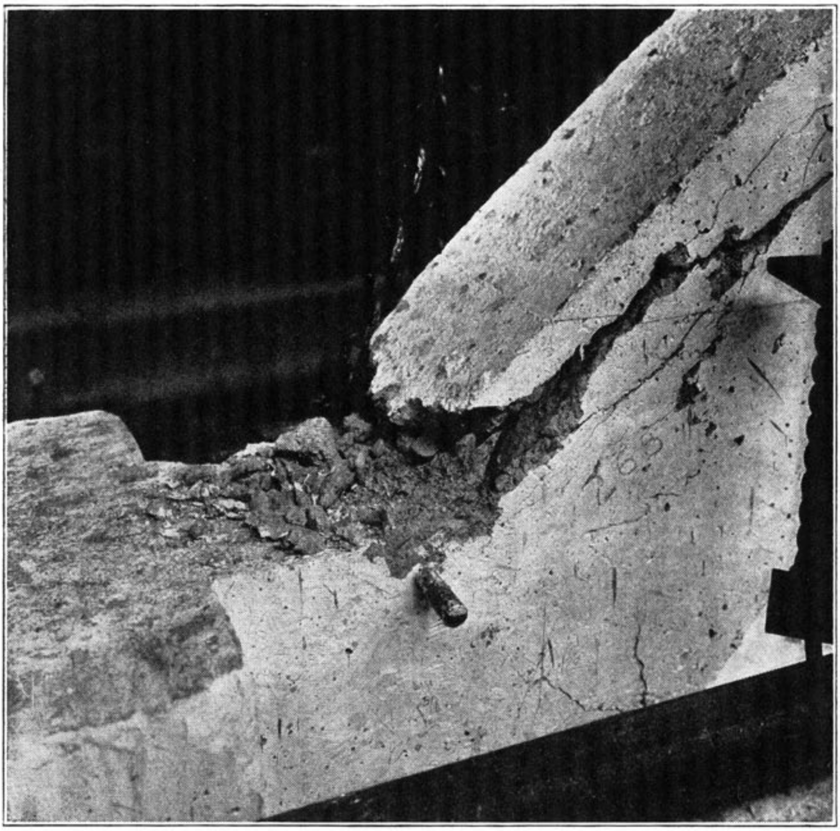


FIG. 19. FAILURE OF ARCH 26-8 AT NORTH SPRINGING DUE TO ROTATION OF ABUTMENTS

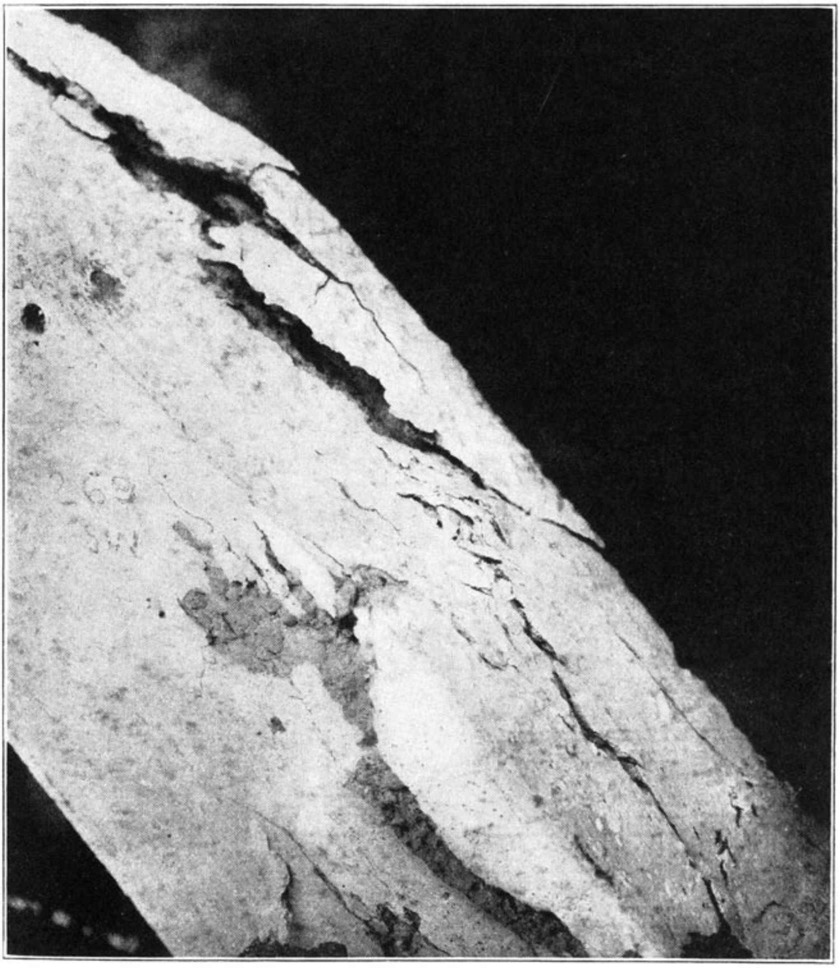


FIG. 20. FAILURE OF ARCH 26-9 AT SOUTH SPRINGING DUE TO ROTATION OF ABUTMENTS

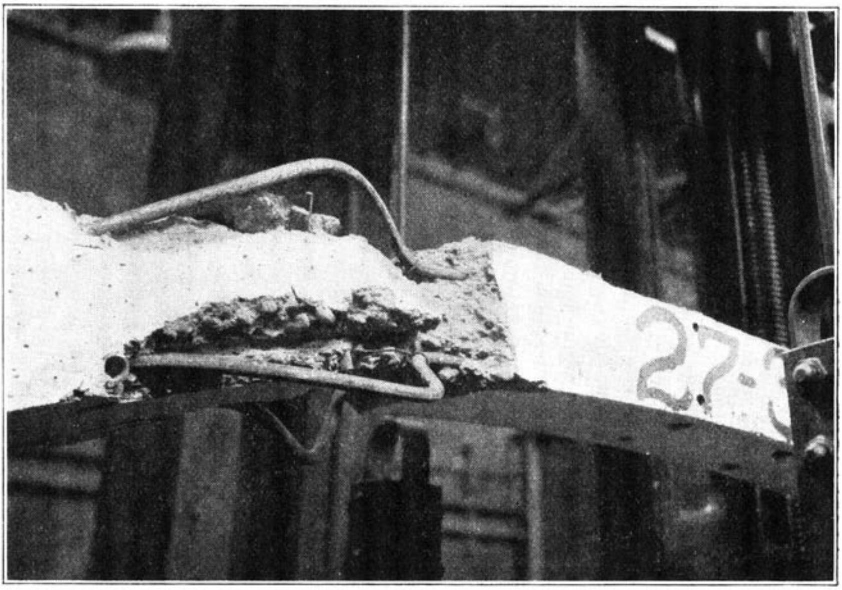


FIG. 21. FAILURE OF ARCH 27-3 AT LOAD-POINT N4

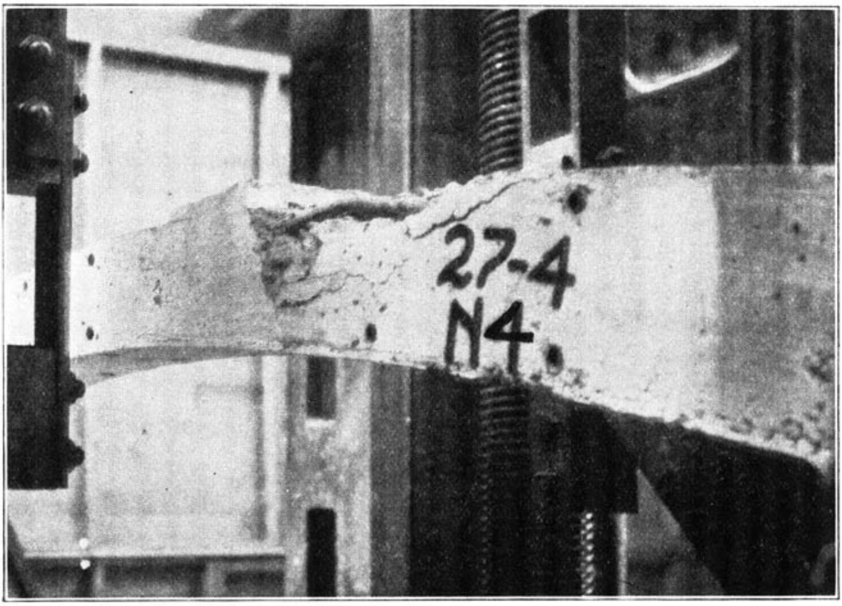


FIG. 22. IMPENDING FLEXURAL FAILURE OF ARCH 27-4 AT LOAD-POINT N4

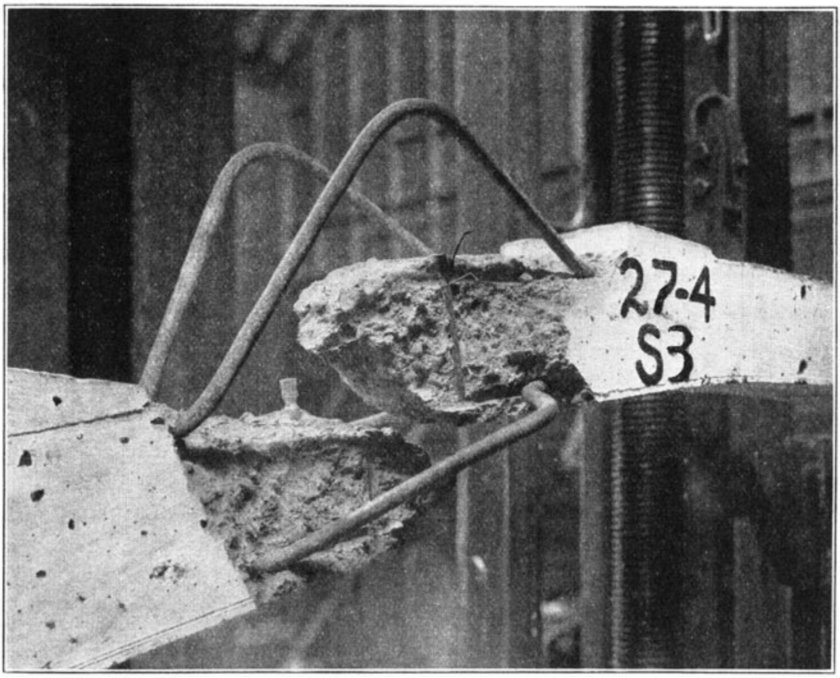


FIG. 23. COMPRESSION FAILURE OF ARCH 27-4 AT SECTION S3

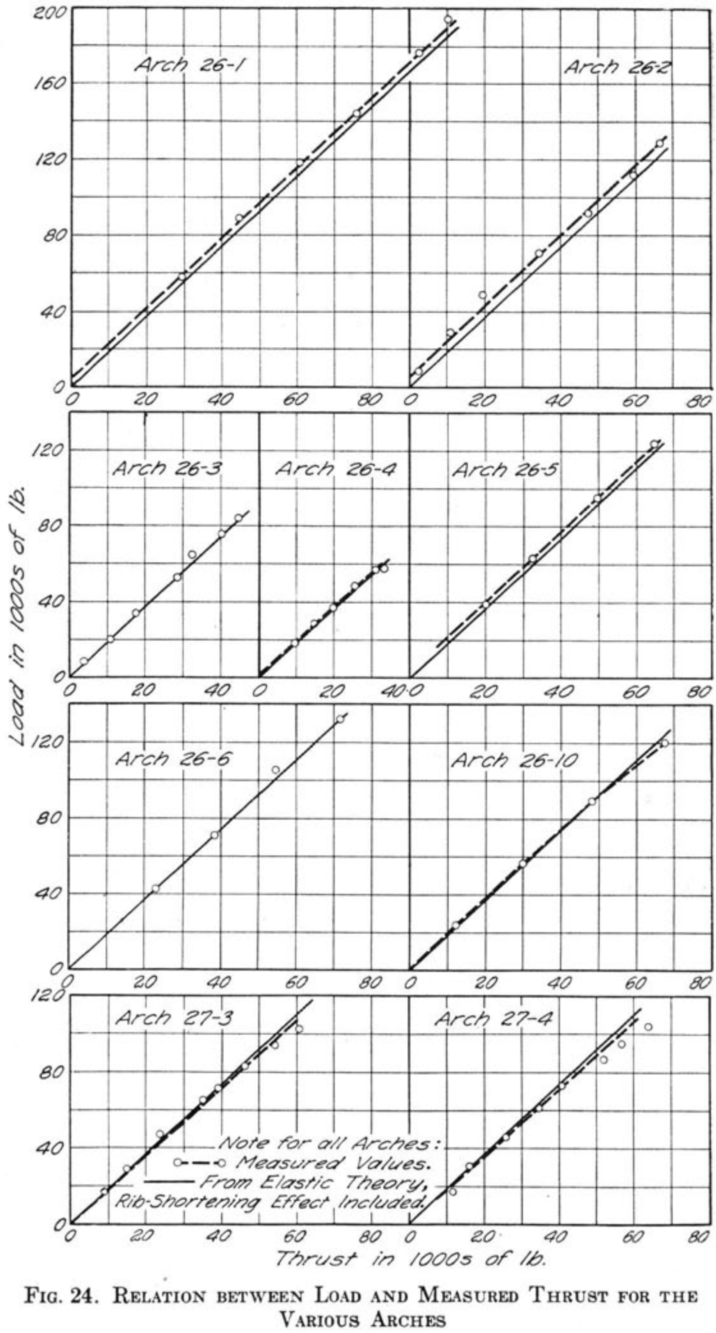


FIG. 24. RELATION BETWEEN LOAD AND MEASURED THRUST FOR THE VARIOUS ARCHES

V. Results of Tests

9. *Relation between Load and Horizontal Thrust.*—The relation between the load and the horizontal component of the abutment reaction was determined for arches 26-1 to 26-6 inclusive, 26-10, 27-3, and 27-4. The thrust was measured with 12-ft. extensometers attached to the tie rods for the arches of the 26 series and by means of helical springs for the arches of the 27 series. These sets of apparatus are described in Section 4, page 21.

A 1000-lb. load, distributed as in these tests, should, according to the elastic theory, produce a horizontal thrust of 542 pounds. The relation between the load and the measured reaction is given in Fig. 24 for the various arches. In this figure the small circles represent observed values, the broken line represents the experimental data in so far as it can be represented by a straight line or a smooth curve, and the light full line represents the values obtained from the elastic theory corrected for rib shortening.

The measured values for arches 26-1 to 26-6, inclusive, are best represented graphically by straight lines and the slope of these lines equals the slope of the theoretical line. The failure of some of the curves to pass through the origin is probably due to an error in the initial reading.

The measured value for arch 26-10 can best be represented by a curved line nearly coincident with the theoretical straight line. The measured values for arches 27-3 and 27-4 have been represented by straight lines slightly below the theoretical curve.

The moment at the crown of the arch is a small difference between two large moments opposite in sense. A considerable part of the positive moment is due to the horizontal component of the abutment reaction and a slight error in this horizontal component causes a large error in the moment. The moment at the crown of the arch is therefore a very sensitive check upon the magnitude of the horizontal abutment reactions. The stress at the top and bottom of the rib was determined at the crown from the measured abutment reactions and also from the strain-gage readings.* A comparison of corresponding values shows that the broken lines of Fig. 24, for arches 26-1 to 26-6, inclusive, lowered so as to pass through the origin, represent thrusts that give stresses at the crown agreeing very closely with the measured values.† In a similar manner it was found that the broken-line diagrams for arches 27-3 and 27-4,‡ shown in Fig. 24,

*See Section 11.
†See Figures 29 to 34, inclusive.
‡See Figures 36 and 37.

give thrusts that are slightly too large. Likewise, the broken-line diagrams of Fig. 24 give thrusts that are slightly too small for arch 26-10 at loads below 90 000 lb. and thrusts that are slightly too large at greater loads.*

These tests apparently indicate that the horizontal thrust on the abutments of each of the experimental arches due to loads distributed in the manner indicated in Fig. 11 can be obtained very accurately if it is computed by the elastic theory, correction being made for rib shortening. This statement is further supported by the discussion in Section 11.

10. *Moment on Abutments Due to Load.*—The moment at the springing where the rib joins the abutment was determined from the loads on the scales under the outer ends of the abutment levers. This moment is reported for arches 26-2, 26-5, 26-6, 26-10, 27-3, and 27-4, all of the arches $6\frac{1}{8}$ in. wide that were tested with fixed abutments. The first four of these arches were subjected to a symmetrical load, and the last two to an unsymmetrical load. The relation between the moments at the springing and the load on the arch is shown in Figs. 25 and 26, the light lines representing the theoretical value of the moment, and the heavy lines the value computed from the measured reactions. The two sets of values are seen to agree fairly well for all arches.

The position of the thrust line at various loads is given in Fig. 27 for the symmetrically loaded arch 26-5, and in Fig. 28 for the unsymmetrically loaded arch 27-4. In these figures the upper diagrams show the position of the thrust line at various loads as determined from the measured abutment reactions, and the lower diagram represents the theoretical position of the thrust line, the same at all loads.

11. *Unit Stress in the Concrete.*—One object of the investigation was to determine the strength developed by the concrete in an arch in order that a comparison might be made with the strength developed by the same concrete in control cylinders. In addition, it was desirable to compare the actual stress at various loads with the theoretical stress at the same loads.

The abutment reactions and the loads at all load-points were known in magnitude, position, and direction from the scale readings

*See Figure 35.

and from the proportions of the levers, and the unit stress at the sections where the strain was measured was determined from these known forces. The relation between the load and the unit stress, as determined from the measured reactions, is shown by the heavy full lines of Figs. 29 to 37, inclusive.

The theoretical unit stress at any section in an arch can be determined from the elastic theory. The theoretical stress at the various sections is given in Table 6 for a 1000-lb. symmetrical load, and in Table 7 for a 1000-lb. unsymmetrical load. These values are based upon a straight-line distribution of combined axial and flexural stress. The relation between the theoretical stress and the load for the various arches is shown by the light dot-and-dash lines of Figs. 29 to 37, inclusive.

Strain-gage readings were taken on the steel and concrete at the top and bottom of the rib at all sections indicated in Fig. 10. Stress can be determined from strain providing the stress-strain relation for the material under consideration is known. The modulus of elasticity of steel is quite constant; for these tests it was assumed to be 30 000 000 lb. per sq. in. The stress-strain relation for concrete, however, is a very uncertain quantity, and its value is so seriously influenced by the manner in which a specimen is poured that strain-gage readings on an arch cannot be satisfactorily interpreted from the stress-strain relation of a control cylinder. Fortunately, however, the tangential thrust can be accurately determined at seven sections of the arch from the measured loads and reactions. This thrust, less the stress taken by the steel, determined from the strain-gage readings on the latter, is the axial load on the concrete. The average unit stress is this axial load divided by the area of the concrete. It is a measured quantity. The mean unit strain was determined at each section from the strain-gage readings at the top and bottom of the rib. In this way the stress-strain relation was determined at each section from the measured stress and strain for the concrete actually at that section. If the strains at the top and bottom, for a given section, are equal, the average stress—mean-strain relation will be the true stress-strain relation; if they vary greatly an error is introduced due to the curvature of the stress-strain diagram for concrete, but even so the strains at the top and bottom are a fair indication of the position of the thrust line. The stress-strain relation for the concrete at various sections of the arches, based upon average stress and mean strain, is given in Figs. 38 to 46, inclusive.

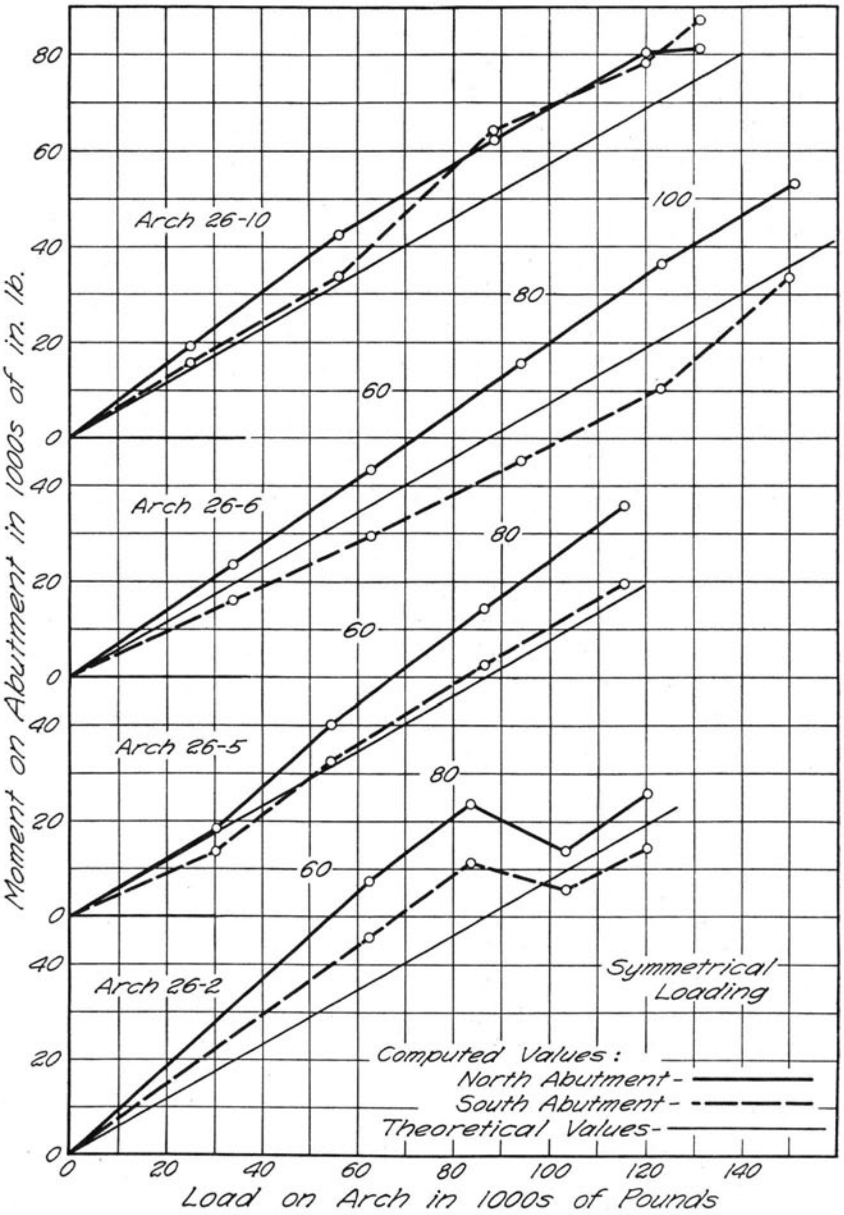


FIG. 25. RELATION BETWEEN LOAD AND MOMENT ON ABUTMENTS, ARCHES 26-2, 26-5, 26-6, AND 26-10

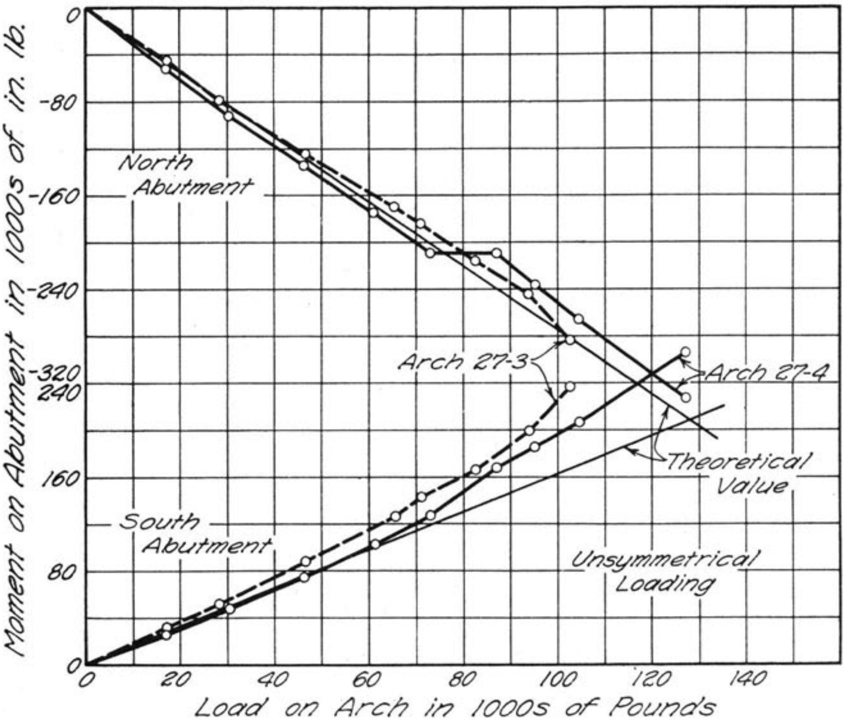


FIG. 26. RELATION BETWEEN LOAD AND MOMENT ON ABUTMENTS, ARCHES 27-3 AND 27-4

The stress obtained from the strain by using the average stress—mean-strain diagrams just mentioned is shown by the broken-line diagrams of Figs. 29 to 37, inclusive. This stress has been designated as the measured stress.

Arches 26-2, 26-5, 26-6, and 26-10 were identical except that the reinforcing was slightly less in 26-2 than in the others. All were subjected to symmetrical loads. The load-stress diagrams determined from the strain-gage readings are remarkably similar at corresponding sections of the various arches. Moreover, the diagrams for corresponding sections at the opposite ends of each arch are nearly identical in practically all cases.

Arches 27-3 and 27-4 were identical, and both were subjected to the unsymmetrical load shown in Fig. 11b, page 27. For these arches, also, the diagrams for corresponding sections are remarkable for their similarity.

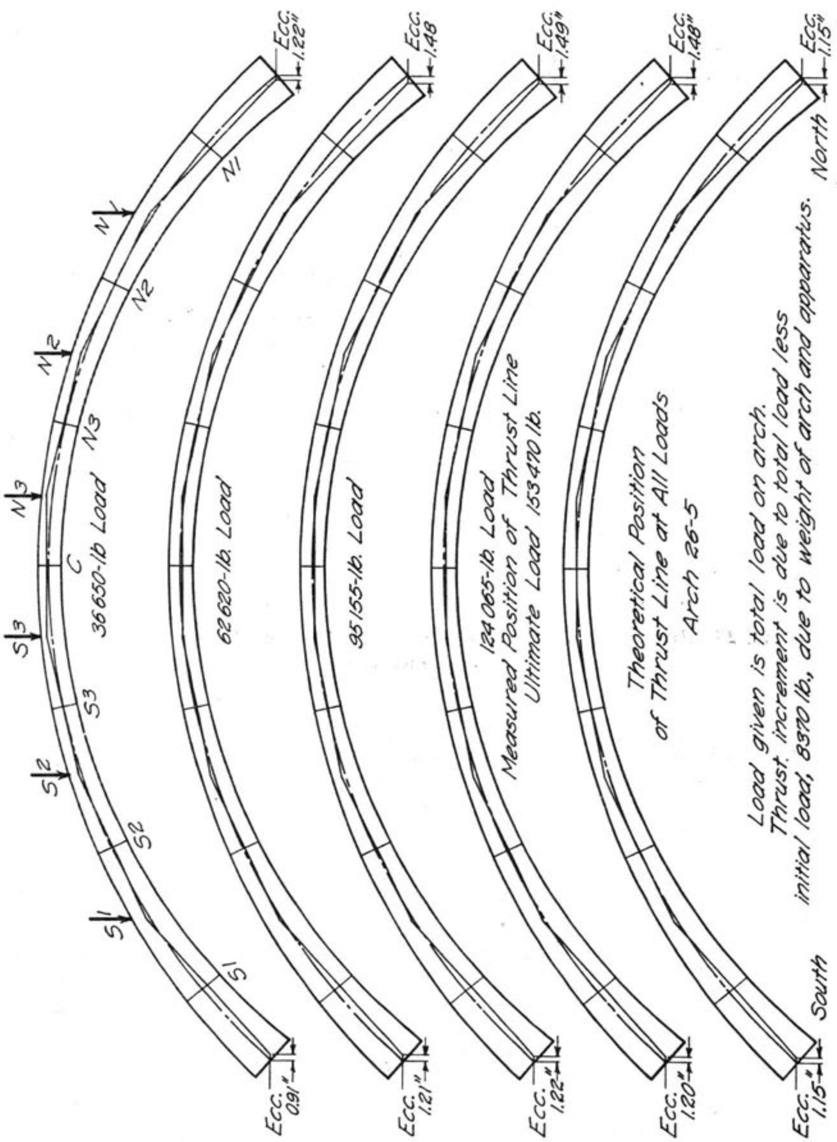


FIG. 27. COMPARISON OF THEORETICAL AND MEASURED POSITION OF THRUST LINE, ARCH 26-5, SYMMETRICALLY LOADED

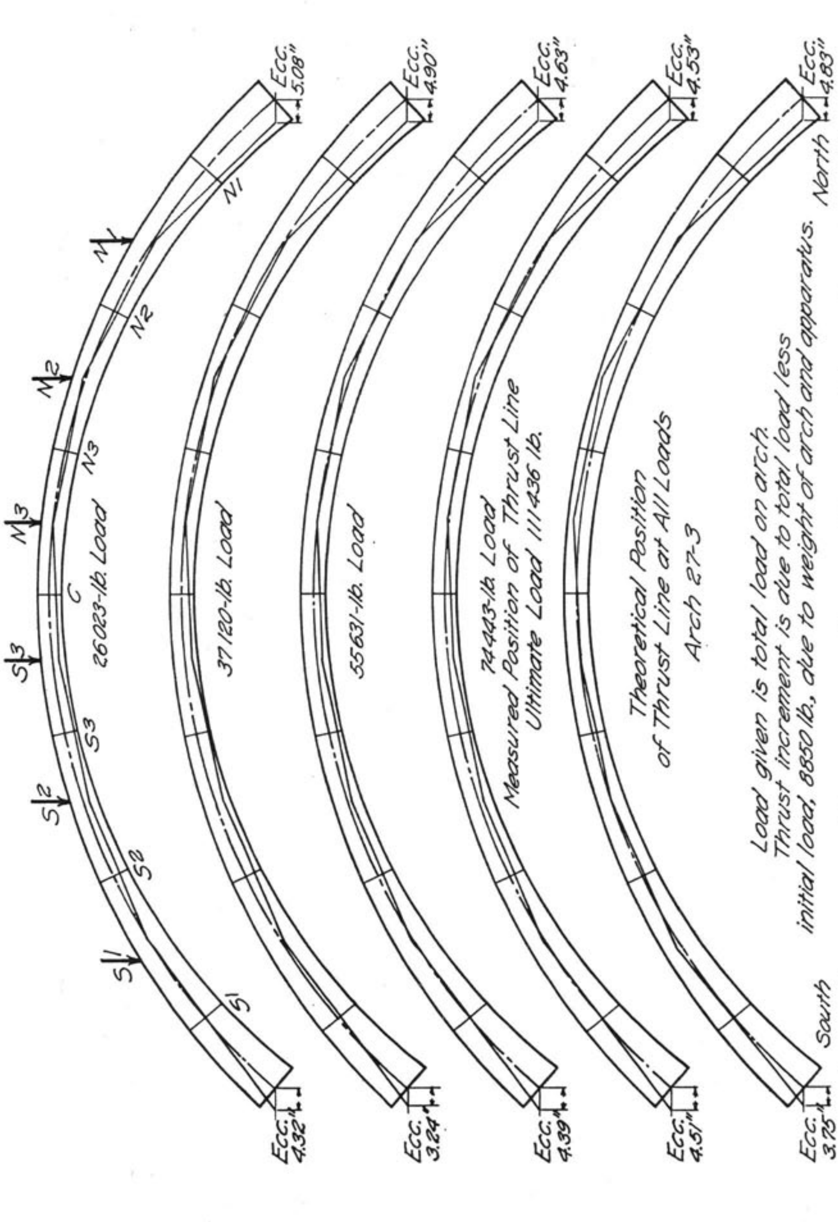


FIG. 28. COMPARISON OF THEORETICAL AND MEASURED POSITION OF THRUST LINE, ARCH 27-3, UNSYMMETRICALLY LOADED

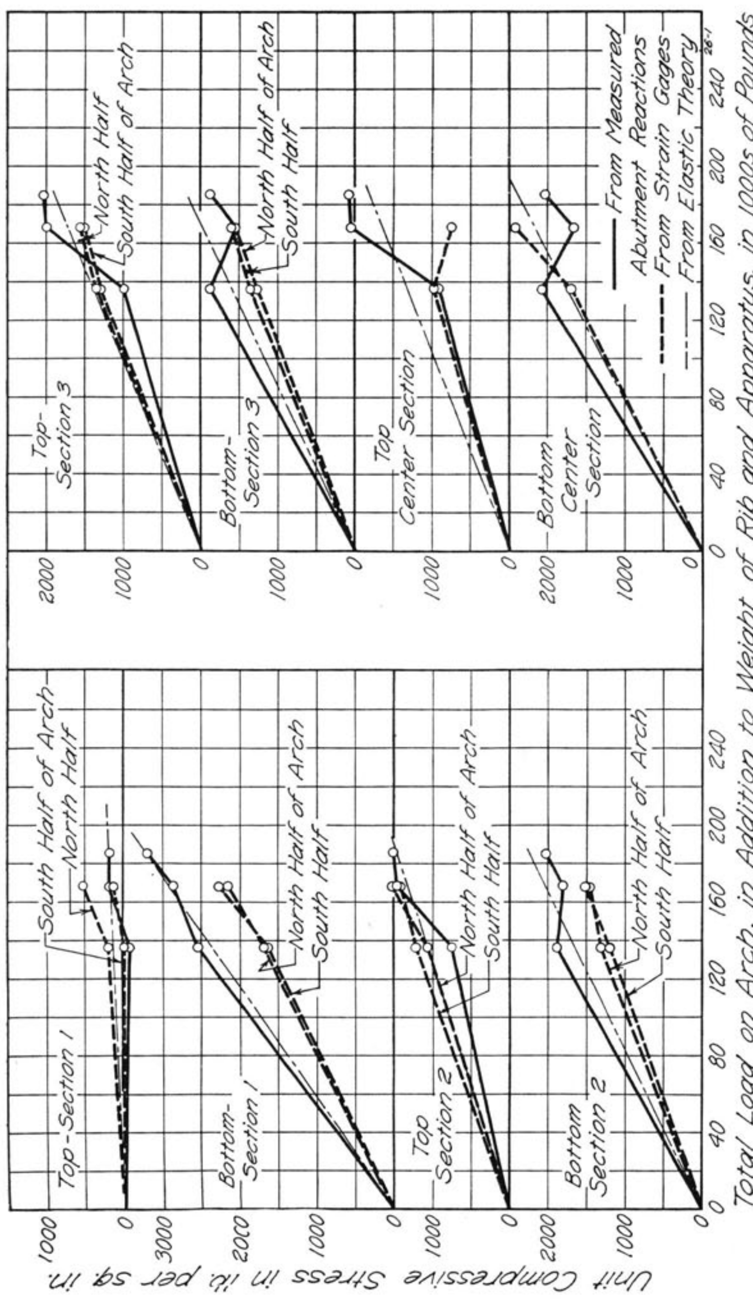


FIG. 29. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 26-1

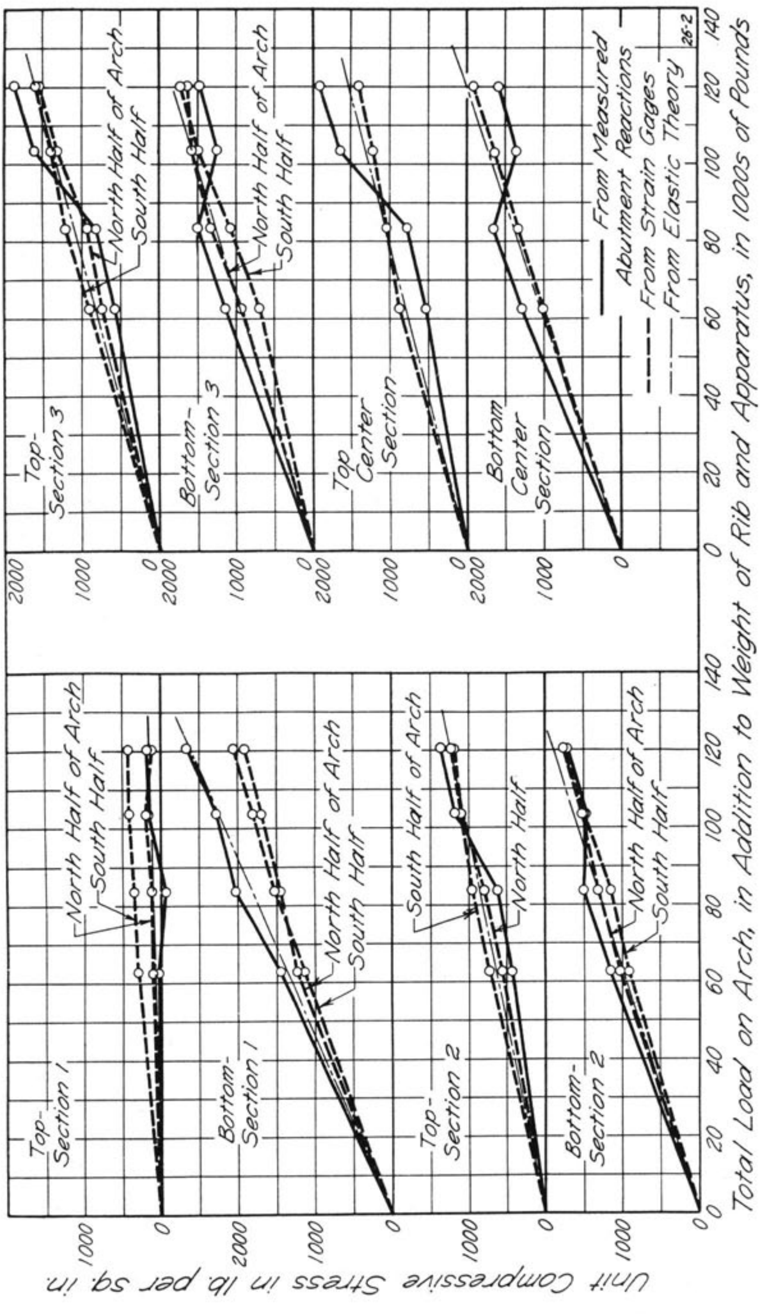


FIG. 30. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 26-2

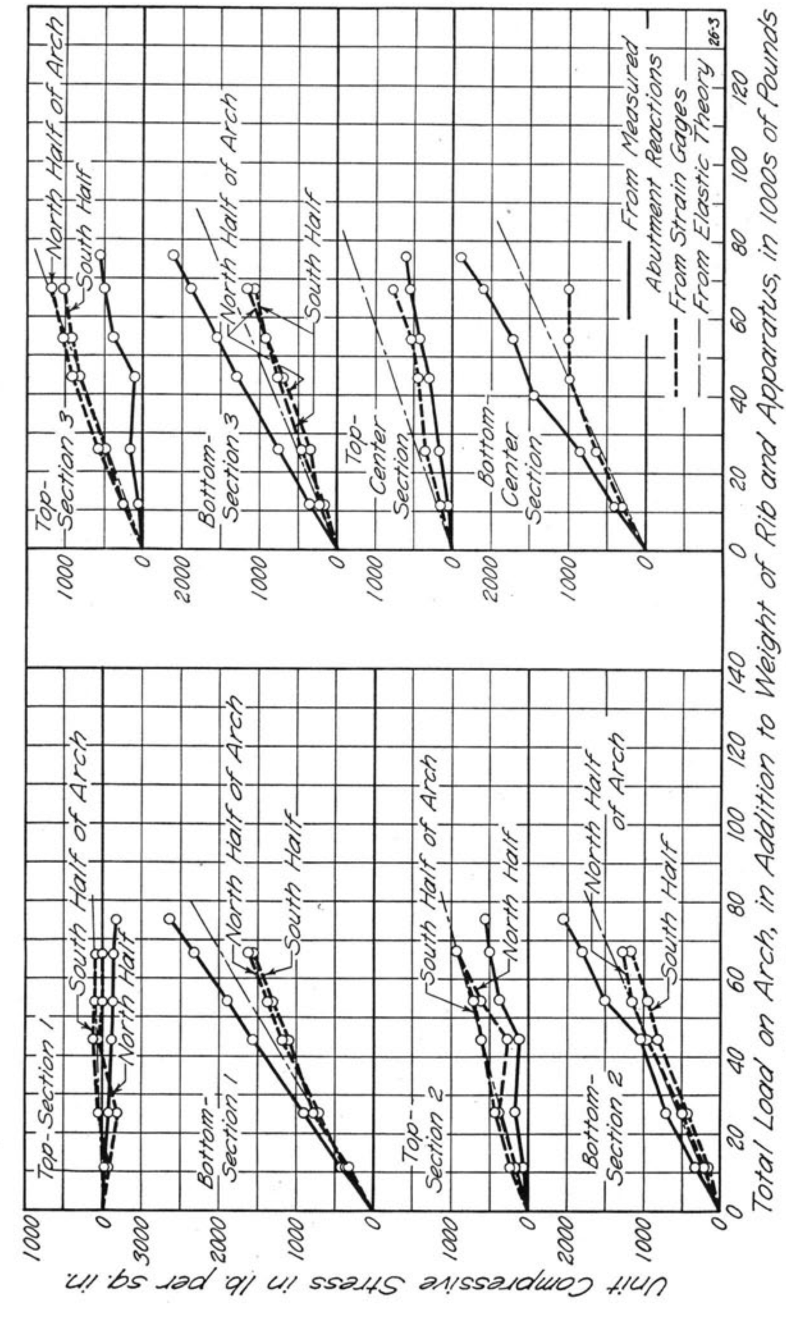


FIG. 31. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 26-3

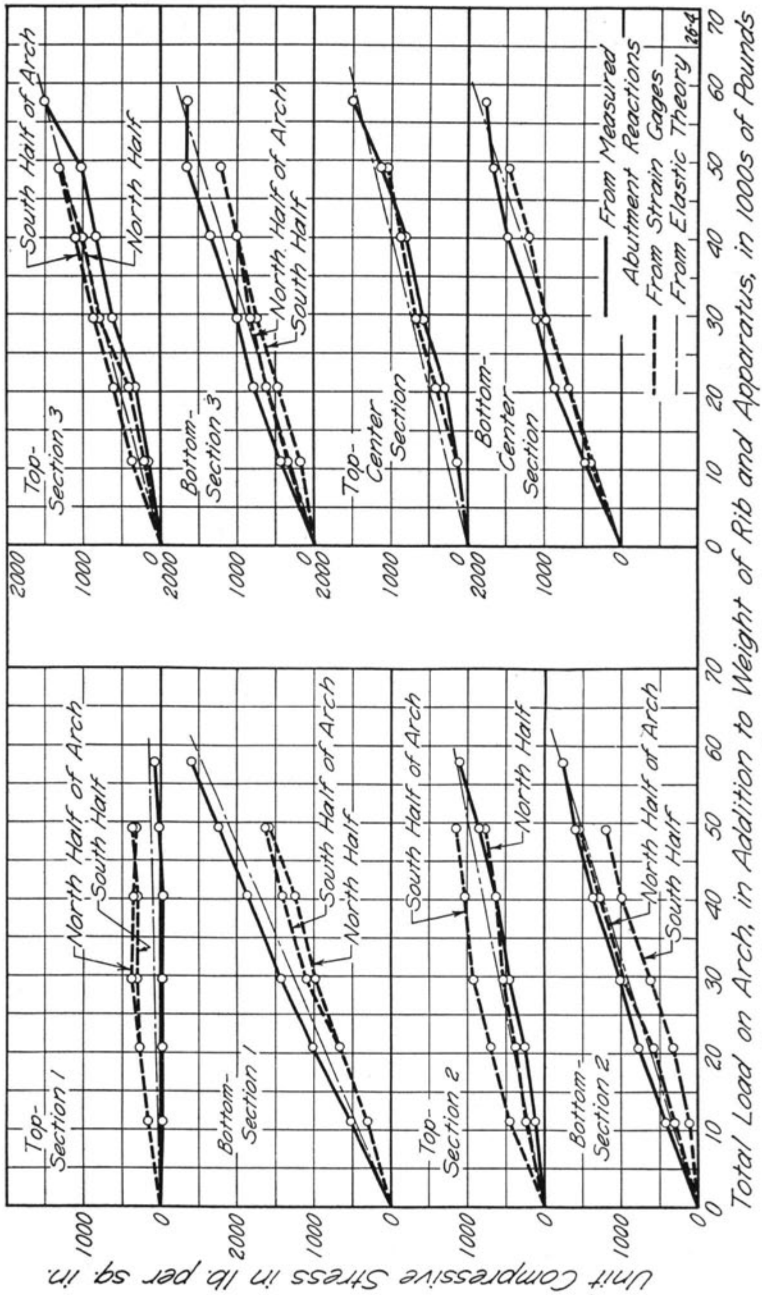


FIG. 32. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 26-4

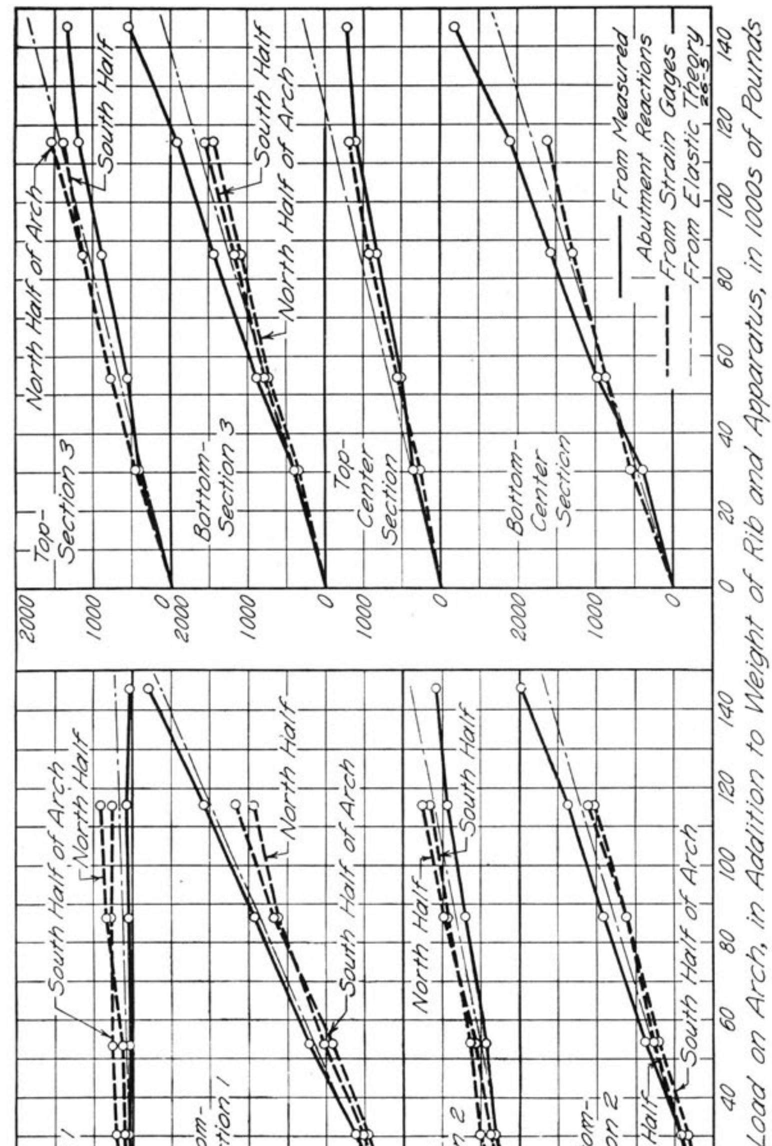


FIG. 33. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 26-5

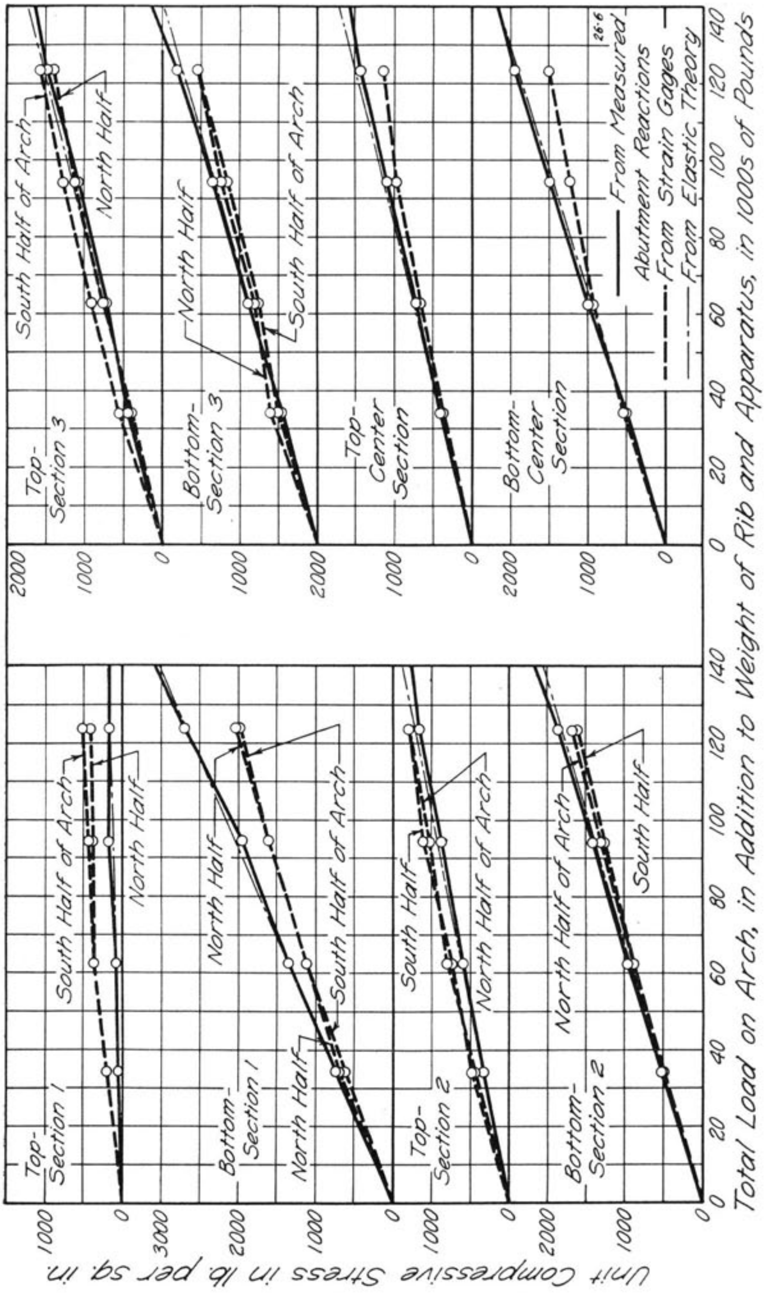


FIG. 34. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 26-6

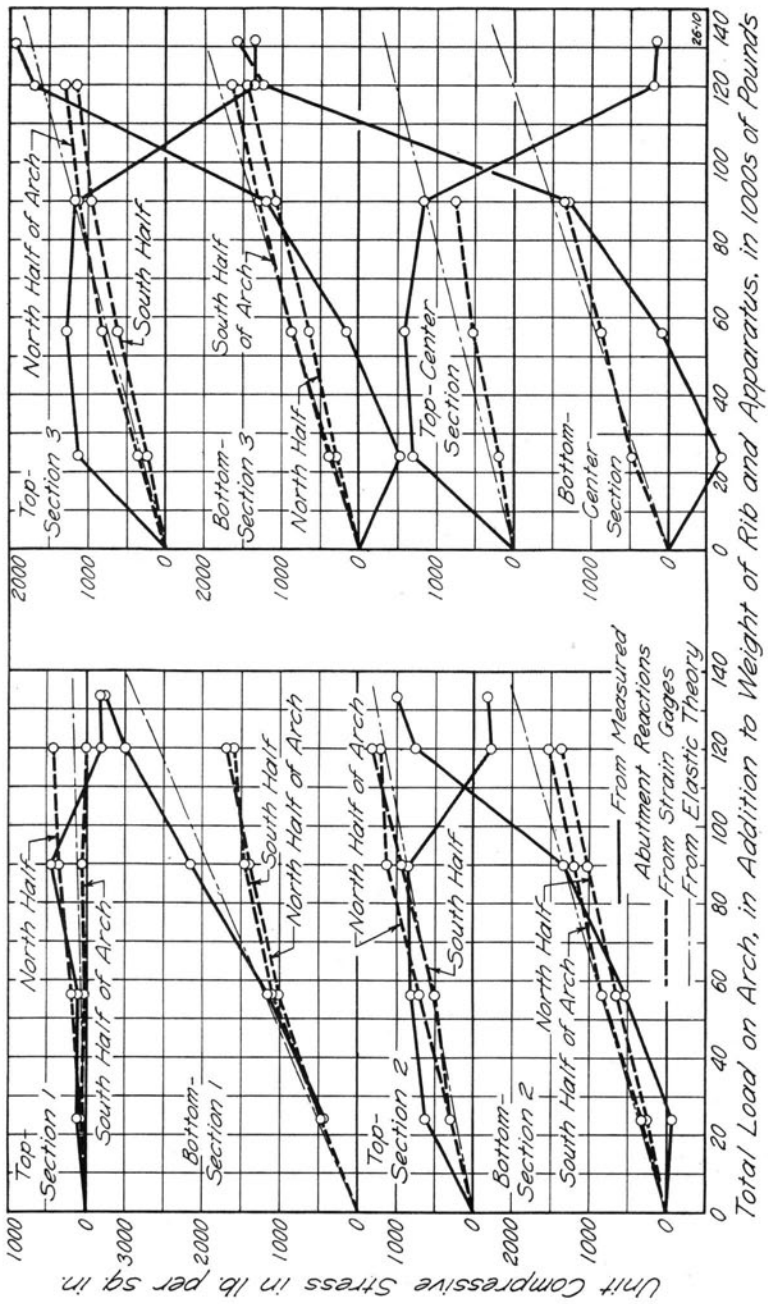


FIG. 35. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 26-10

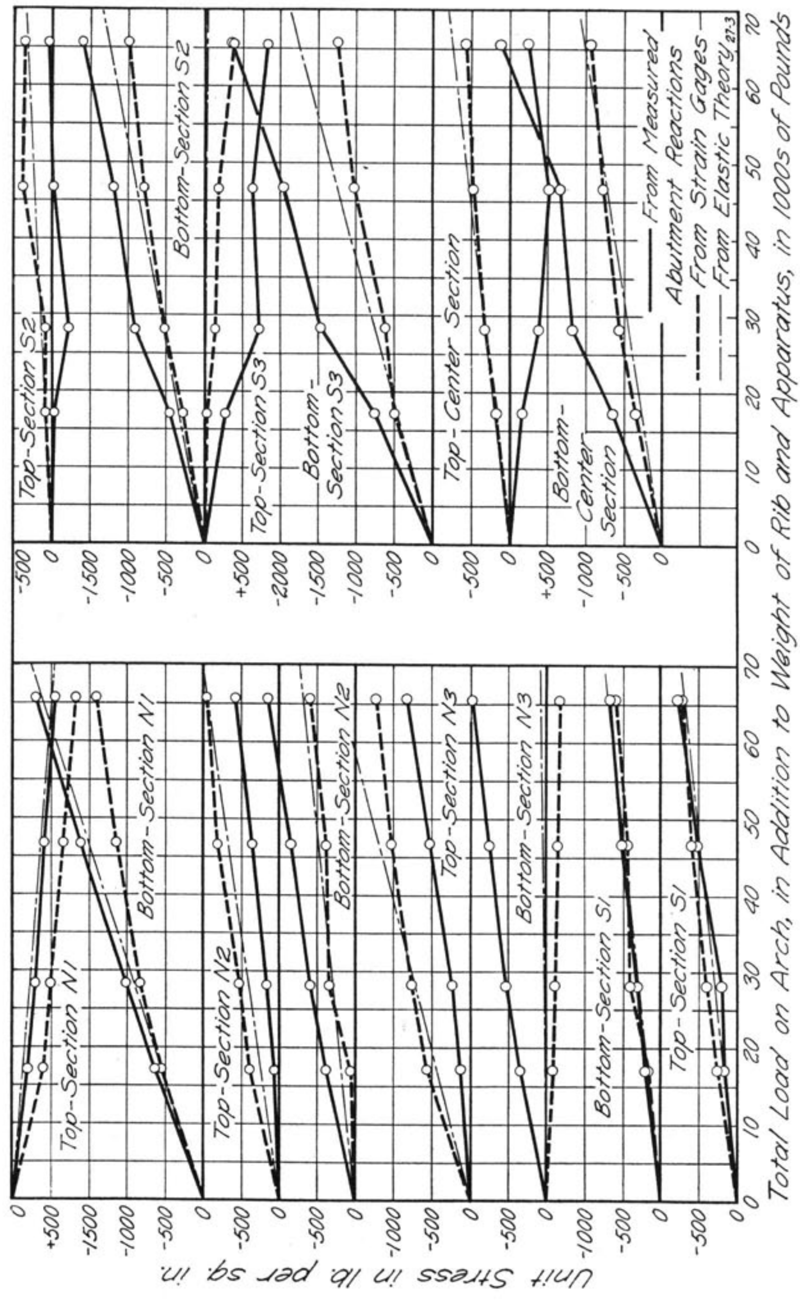


FIG. 36. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 27-3

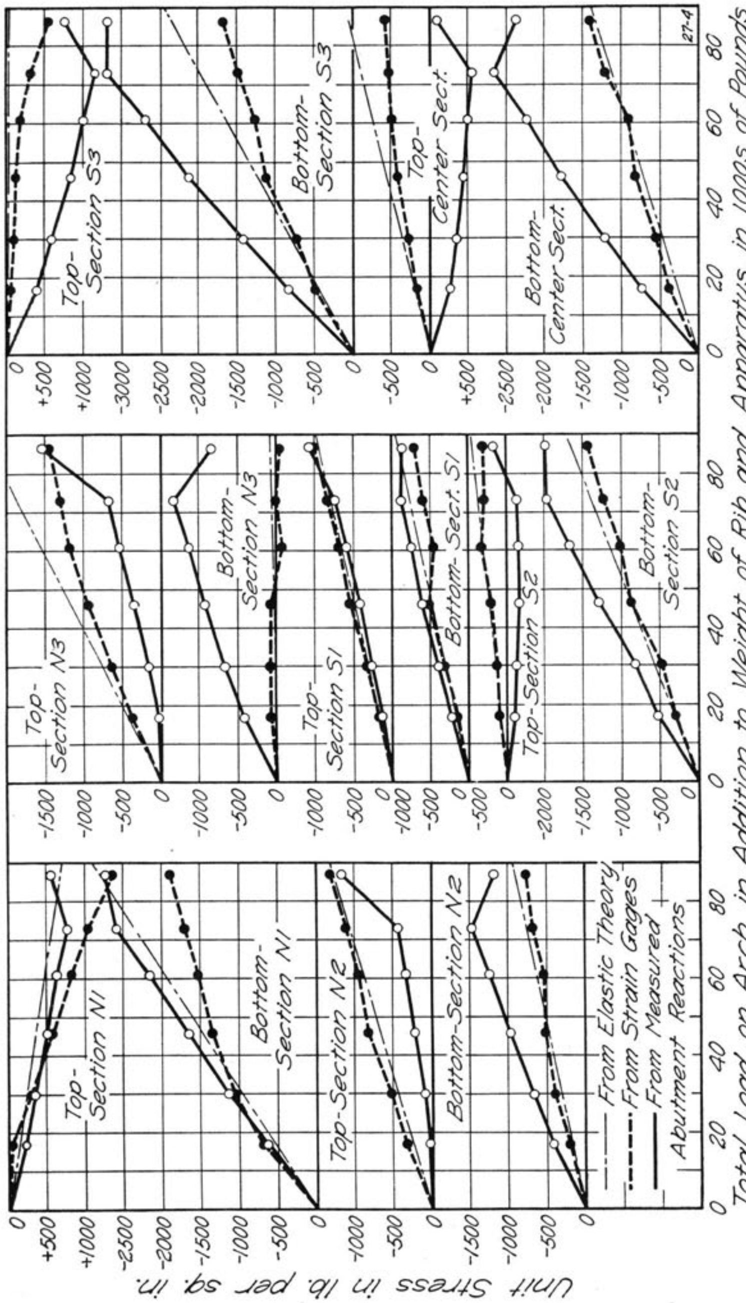


FIG. 37. RELATION BETWEEN LOAD AND UNIT STRESS IN CONCRETE, ARCH 27-4

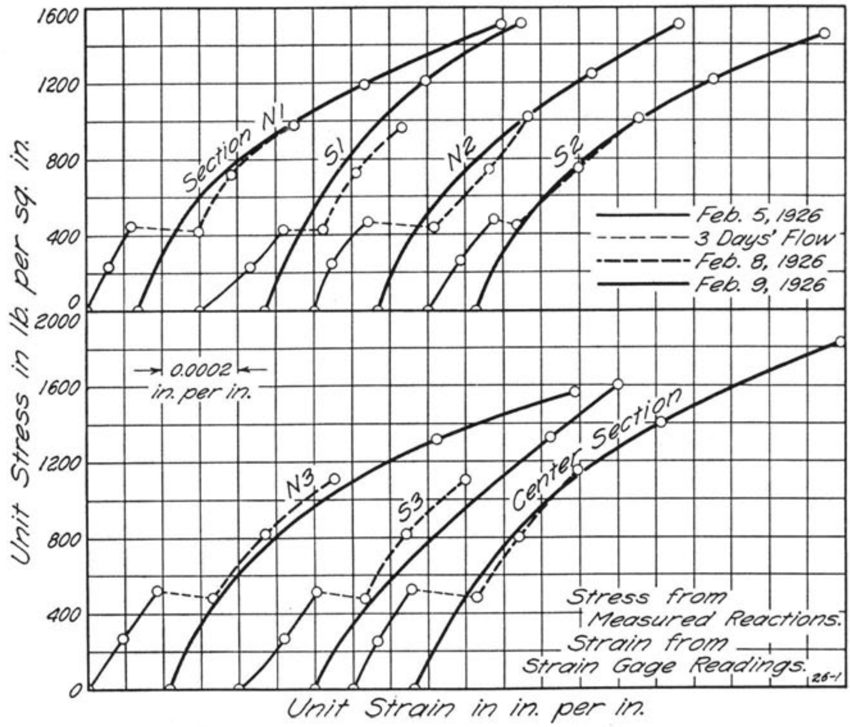


FIG. 38. AVERAGE-STRESS—MEAN-STRAIN RELATION FOR CONCRETE, ARCH 26-1

The theoretical and measured values of the stress are compared in Figs. 47 to 51, inclusive, the first three figures being for symmetrically loaded arches, and the last two for unsymmetrically loaded arches. Similar diagrams for the other arches would show about the same relation between two values of the stress. A comparison of the two values can also be obtained by noting the relative positions of the light dot-and-dash lines and the heavy broken lines of Figs. 29 to 37.

The ratio of the theoretical to the measured value of the stress at the various sections of the arches is given in Table 23 for symmetrically loaded and in Table 24 for unsymmetrically loaded arches, the values given being for the maximum load at which readings were taken for each arch. In Table 23 values for individual arches are given in columns 2 to 8, and the average for all of the arches is given in column 9; the ratio of the flexural to the axial stress is given in column 12. Corresponding information for unsymmetrically

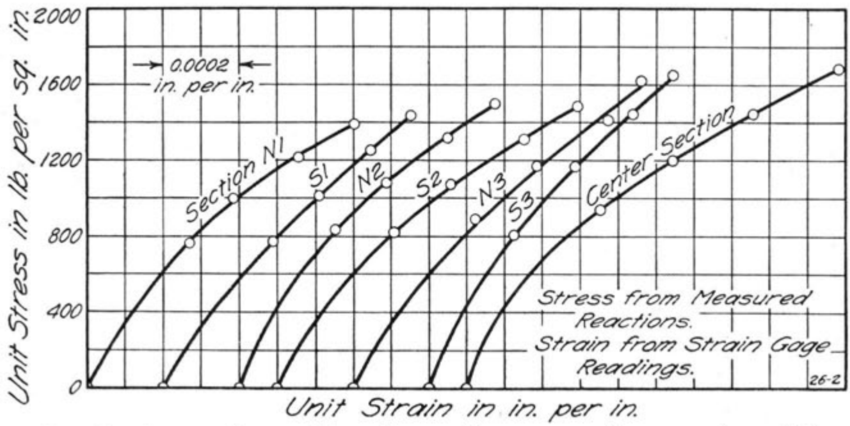


FIG. 39. AVERAGE-STRESS—MEAN-STRAIN RELATION FOR CONCRETE, ARCH 26-2

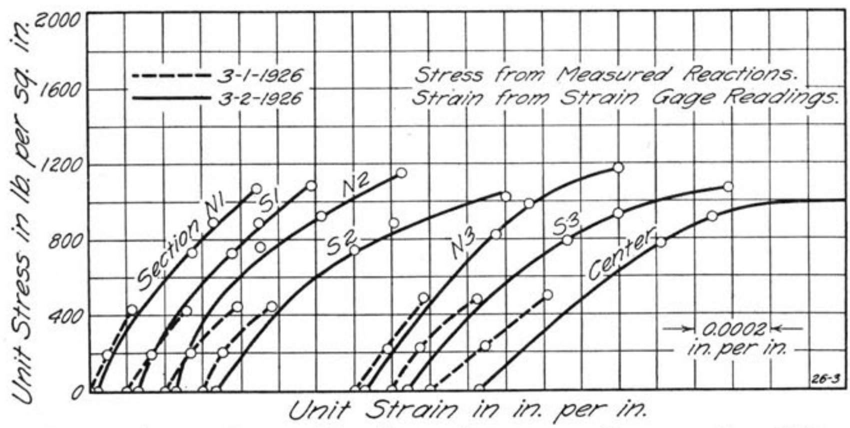


FIG. 40. AVERAGE-STRESS—MEAN-STRAIN RELATION FOR CONCRETE, ARCH 26-3

loaded arches is given in the various columns of Table 24. An inspection of these tables shows that at sections where the flexural stress is small the measured and theoretical values agree very well, but at sections having relatively large flexure, the measured stress is much less than the theoretical stress. Part of this discrepancy is due to the manner in which the stress was determined. Two inaccuracies are known to have been introduced, one in the determination of the theoretical and the other in the measured stress. Converting strain into stress by the use of an average stress—mean-strain diagram in place

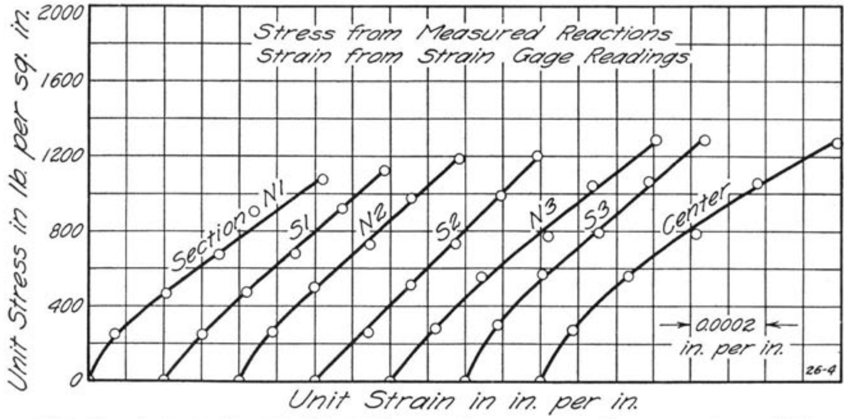


FIG. 41. AVERAGE-STRESS—MEAN-STRAIN RELATION FOR CONCRETE, ARCH 26-4

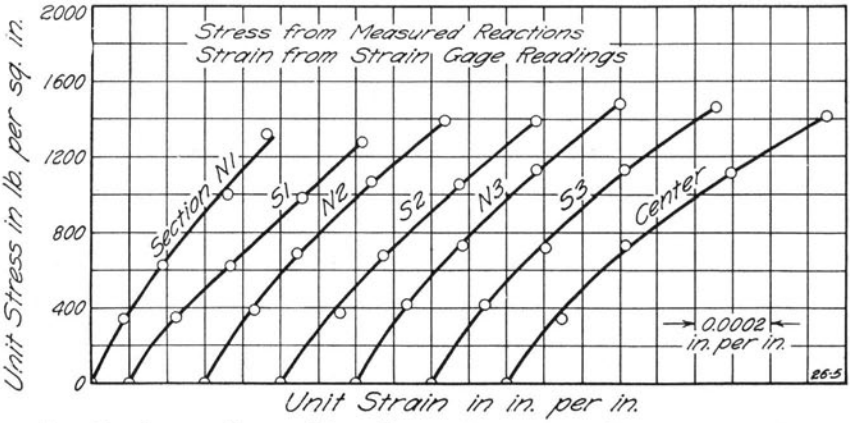


FIG. 42. AVERAGE-STRESS—MEAN-STRAIN RELATION FOR CONCRETE, ARCH 26-5

of a true stress-strain diagram gives values of the stress that are too small at sections subjected to a large moment, due to the fact that stress is not proportional to strain. This error may have a value of 10 per cent if the flexural stress equals the axial stress and if the concrete is stressed to two-thirds of its ultimate strength. The second error is also due to the fact that stress is not proportional to strain. In computing the stress due to a given moment, the stress is assumed to vary across the section proportionally to the ordinates of a straight line whereas the stress really varies more nearly as the ordinates of

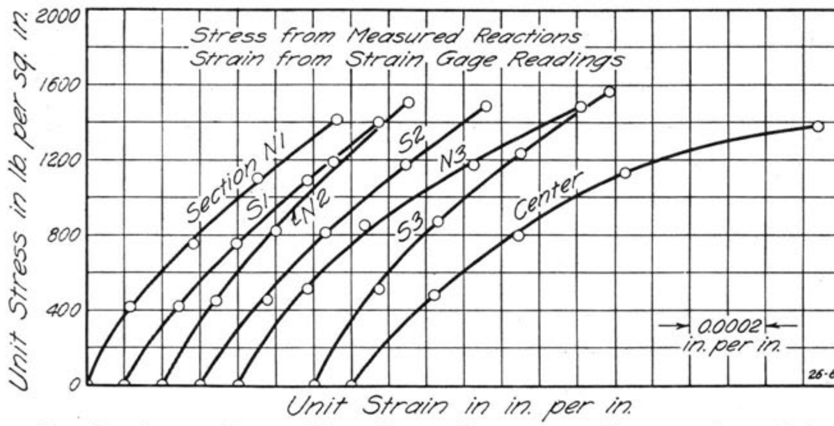


FIG. 43. AVERAGE-STRESS—MEAN-STRAIN RELATION FOR CONCRETE, ARCH 26-6

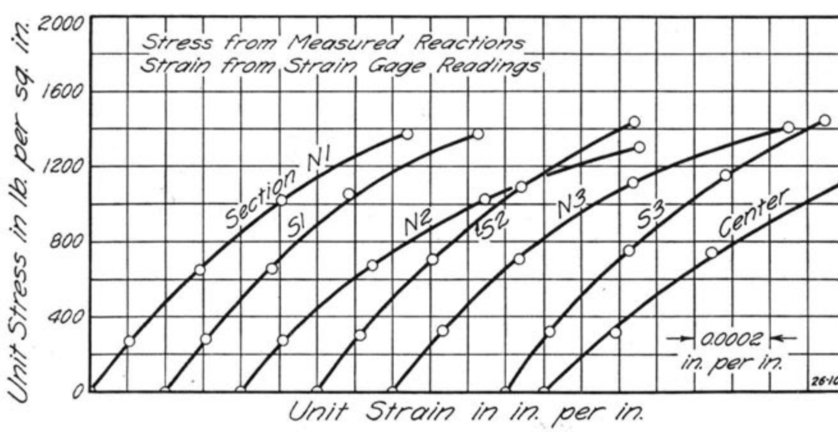


FIG. 44. AVERAGE-STRESS—MEAN-STRAIN RELATION FOR CONCRETE, ARCH 26-10

a parabola. Consequently, the computed stress at the surface of the rib due to a given moment is a little greater than the true stress. That is, of the two values of the stress given, the measured and the computed, the measured stress is a little too small due to an inaccuracy in the diagram by means of which the strains are converted into stresses; and the computed stress is a little too large due to the manner of computing unit stress due to a given moment. Both of these inaccuracies increase with an increase in the flexural stress but the two together will hardly account for all of the discrepancy be-

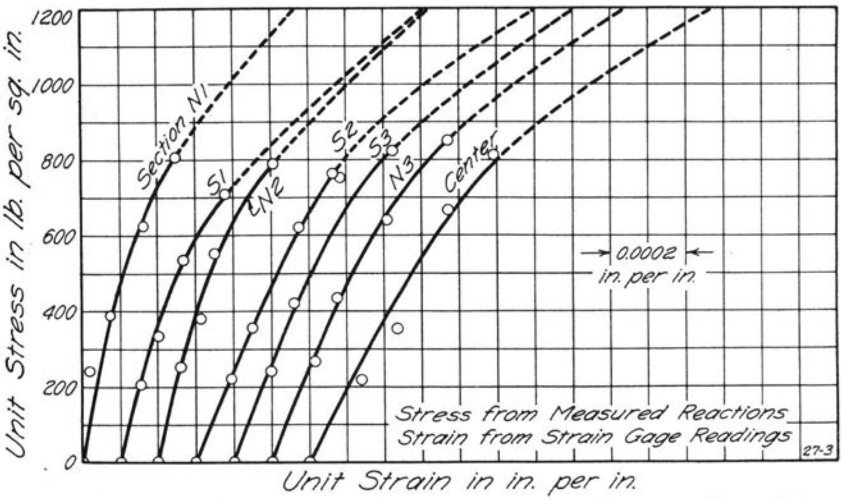


Fig. 45. Average-Stress—Mean-Strain Relation for Concrete, Arch 27-3

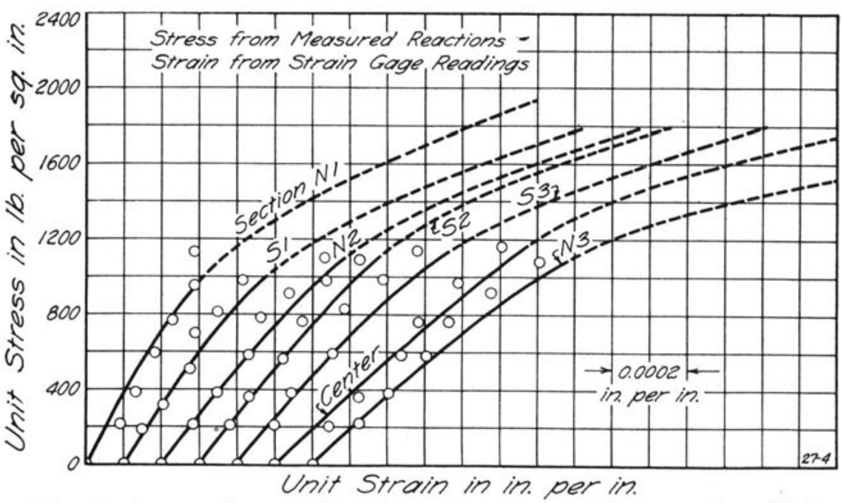


Fig. 46. Average-Stress—Mean-Strain Relation for Concrete, Arch 27-4

tween the measured and the computed values. The resultant discrepancy is no larger, however, than is usually encountered in experimental work. On the whole the tests would seem to indicate that the measured and computed values of the stress agree fairly closely for all sections for which the thrust is within the kern.

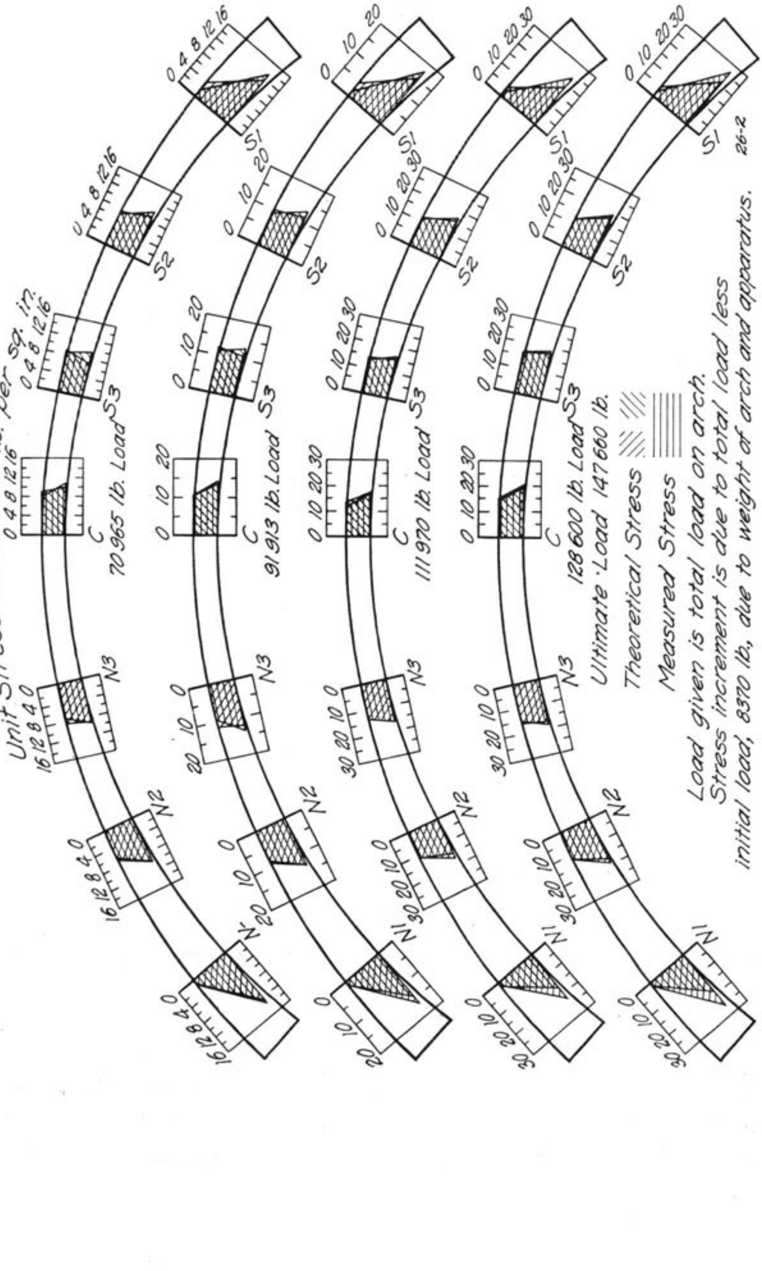


FIG. 47. COMPARISON OF THEORETICAL AND MEASURED STRESS, ARCH 26-2

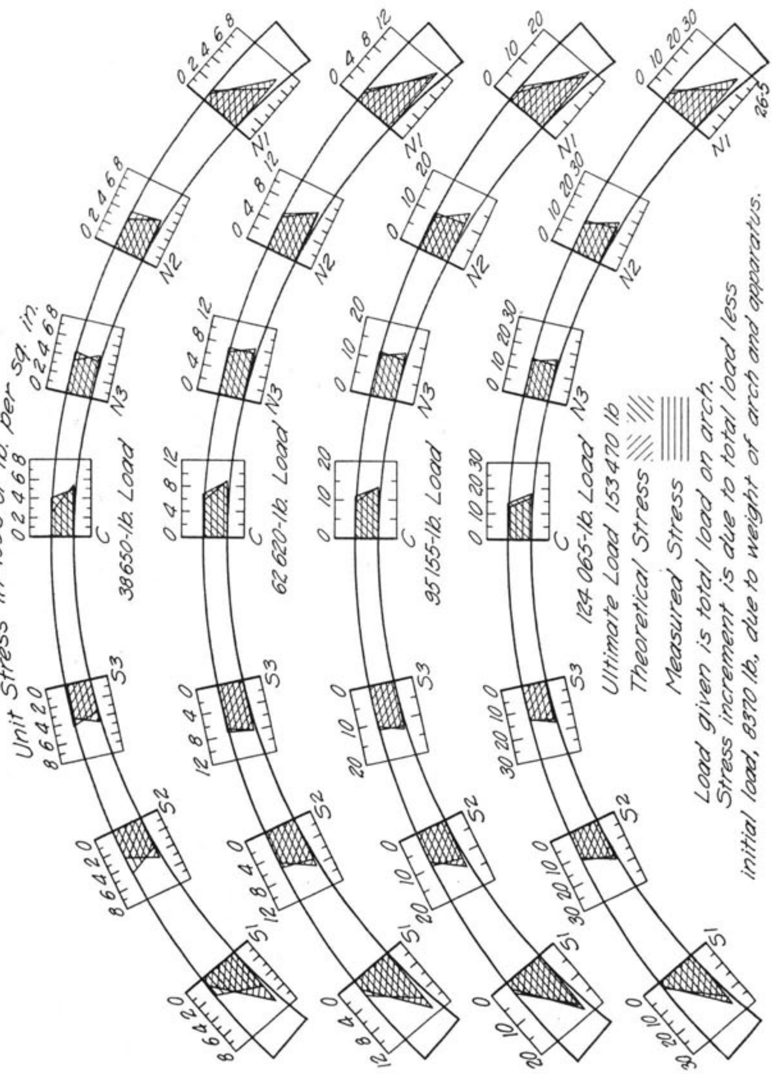


FIG. 48. COMPARISON OF THEORETICAL AND MEASURED STRESS, ARCH 26-5

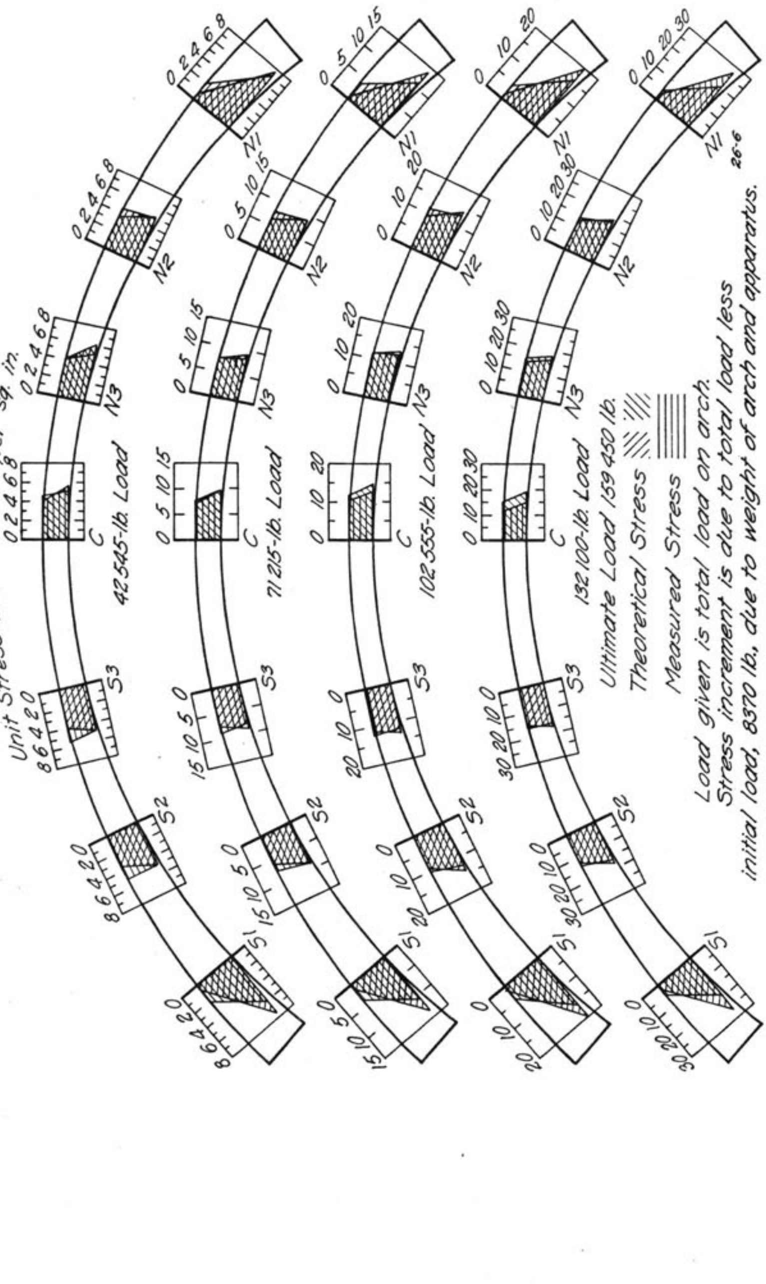


FIG. 49. COMPARISON OF THEORETICAL AND MEASURED STRESS, ARCH 26-6

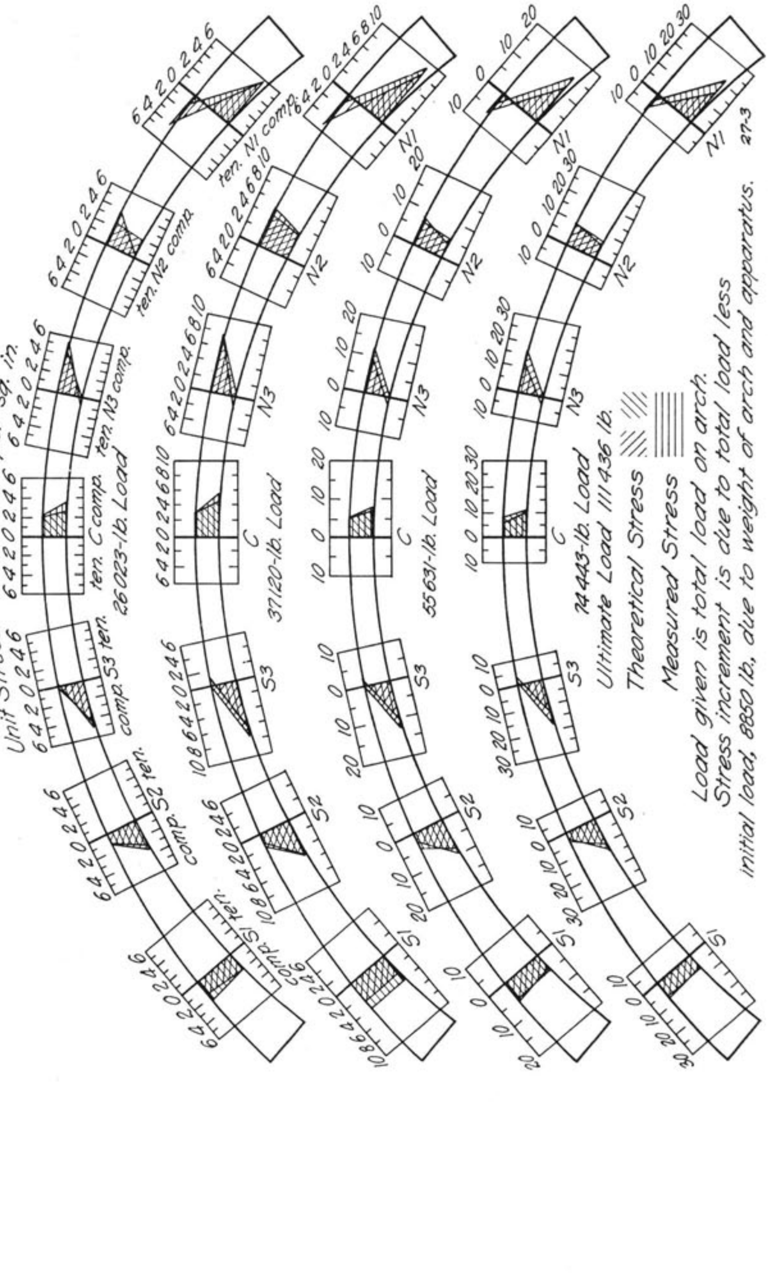


FIG. 50. COMPARISON OF THEORETICAL AND MEASURED STRESS, ARCH 27-3

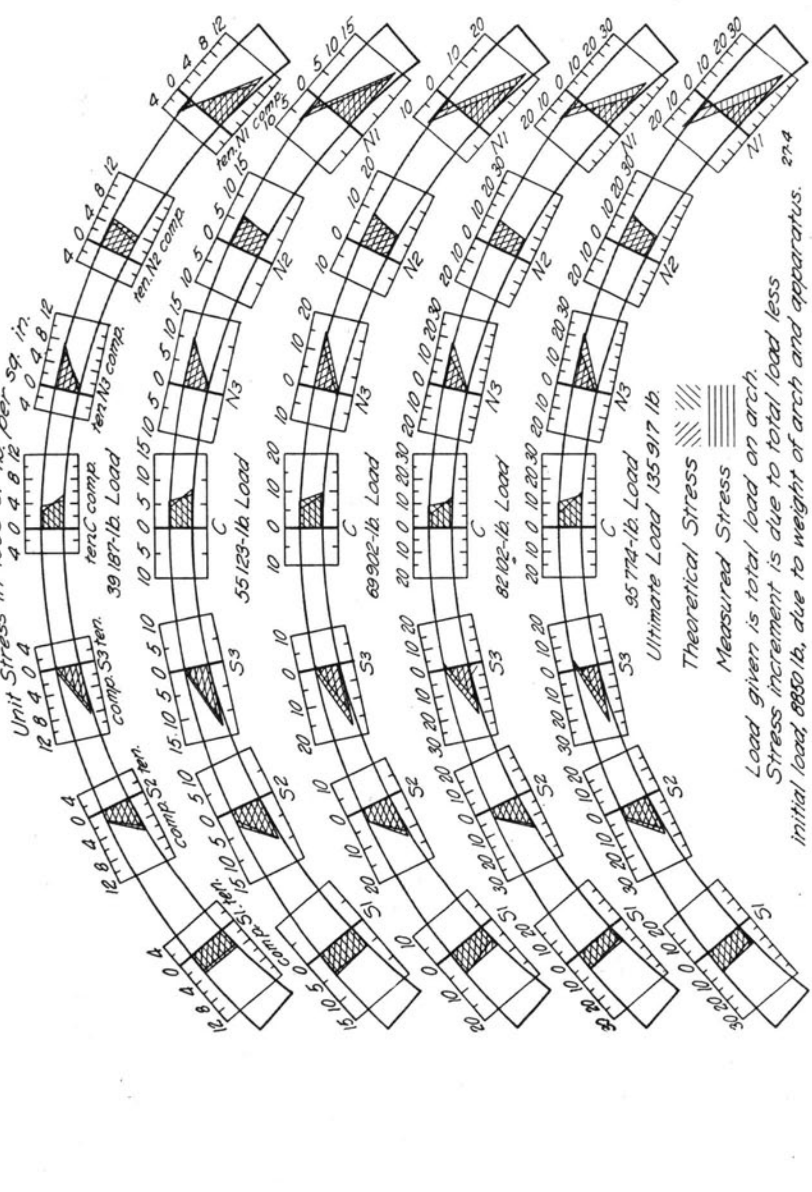


FIG. 51. COMPARISON OF THEORETICAL AND MEASURED STRESS, ARCH 27-4

TABLE 23

RELATION BETWEEN RATIO OF FLEXURAL TO AXIAL STRESS ON A SECTION AND RATIO OF THEORETICAL TO MEASURED STRESS ON SAME SECTION FOR SYMMETRICALLY LOADED ARCHES

Section No.	Ratio of Theoretical to Measured Stress								Theoretical Stress due to 1000-lb. Load		Ratio Flexural to Axial Stress
	Arch No.										
	26–1	26–2	26–3	26–4	26–5	26–6	26–10	Av.	Top	Bottom	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
N1 and S1..............	1.36	1.37	1.35	1.35	1.14	1.32	1.47	1.337	− 1.5	−21.5	0.87
N2 and S2..............	1.34	1.09	1.21	0.98	1.09	1.11	1.22	1.149	− 9.6	−14.8	0.21
N3 and S3..............	1.22	1.07	1.20	1.10	1.12	1.14	1.18	1.147	−12.4	−14.2	0.07
Center..................	1.00	1.06	1.60	1.10	1.11	1.27	1.04	1.168	−12.0	−15.6	0.15

TABLE 24

RELATION BETWEEN RATIO OF FLEXURAL TO AXIAL UNIT STRESS ON A SECTION AND RATIO OF THEORETICAL TO MEASURED STRESS ON SAME SECTION FOR UNSYMMETRICALLY LOADED ARCHES

Section No.	Ratio of Theoretical to Measured Stress			Theoretical Stress Due to 1000-lb. Load		Ratio Flex. to Axial Stress
	Arch No.			Top	Bottom	
	27–3	27–4	Average			
(1)	(2)	(3)	(4)	(5)	(6)	(7)
N1	1.53	1.43	1.48	+ 7.9	−32.2	1.64
N2	0.99	0.97	0.98	−14.5	−10.3	0.17
N3	1.38	1.47	1.43	−25.7	− 0.9	0.93
Center	1.06	0.94	1.00	−12.2	−15.4	0.12
S3	1.42	1.38	1.40	+ 0.4	−27.0	1.03
S2	1.30	1.13	1.22	− 5.0	−19.0	0.58
S1	1.02	0.94	0.98	−11.0	−10.6	0.02

The fact that the agreement between the measured and the computed values is so excellent for sections subjected to little flexure indicates that the actual position of the thrust line is very close to the theoretical position. This really is the best criterion of the agreement between theoretical and measured values of the stress, for the magnitude of measured and computed values of the thrust are in almost perfect agreement. Since the theoretical and actual position of the thrust line agree at sections where the measured quantities can be determined with the greatest accuracy, the thrust line cannot be far from its true position at adjacent sections where the flexure is greater.

The measured stress at the crown of a rib is of interest because of its relation to the horizontal component of the thrust at the springing. The unit stress in an arch is made up of two parts, one due to the thrust and the other to the moment. The resultant moment at the crown of the arch is the small difference between two large moments opposite in sense, and a small error in one of these moments makes a large error in the resultant and in the unit stress. For example, a load on the arch of 100 000 lb. produces a horizontal thrust of 54 200 lb. and an error of one per cent in this thrust will produce an error in flexural stress at the crown of 1015 lb. per sq. in. Table 6 shows that the total stress, combined axial and flexural, on the bottom of the rib at the center due to a symmetrical load of 100 000 lb. is 1560 lb. per sq. in. so that an error of one per cent in the thrust produces an error of 65 per cent in the unit stress at the crown.

TABLE 25

ULTIMATE LOAD AND MAXIMUM THEORETICAL UNIT STRESS FOR SYMMETRICALLY LOADED ARCHES

Arch No.	Width of Rib in.	Point of Failure	Ultimate Load lb.	Section No.	Unit Stress at Ultimate Load by Elastic Theory in lb. per sq. in.
(1)	(2)	(3)	(4)	(5)	(6)
26-1	8	Center	193 820	N1 and S1 N2 and S2 N3 and S3 Center	3396 2351 2254 2479*
26-2	6⅛	Load-Pt. S2	153 110	N1 and S1 N2 and S2 N3 and S3 Center	3494 2428* 2318 2557
26-3	4½	Center	84 800	N1 and S1 N2 and S2 N3 and S3 Center	2636 1834 1744 1942*
26-4	3⅛	Center	65 700	N1 and S1 N2 and S2 N3 and S3 Center	2936 2032 1952 2169*
26-5	6⅛	Center	153 470	N1 and S1 N2 and S2 N3 and S3 Center	3300 2271 2179 2394*
26-6	6⅛	Center	159 450	N1 and S1 N2 and S2 N3 and S3 Center	3428 2360 2264 2487*
26-10	6⅛	Load-Point S4	147 860	N1 and S1 N2 and S2 N3 and S3 Center	3179 2188 2100 2307*

*Section where failure occurred.

Figure 24 shows that the experimental load-thrust diagrams from which the thrusts used in the calculated stresses were read are straight lines having the same slope as the theoretical diagram. These diagrams are for the arches for which the two values of the stress, one from the measured abutment reactions and the other from the elastic theory, are in close agreement. That is, the measured stress indicates that the measured reactions are correct.

Figure 24 shows the load-thrust line for arch 26-10 to be a curved line lying slightly above the theoretical diagram up to a load of 95 000 lb. and below that line for larger loads. Figure 35 shows that the difference between the value of the stress computed by the elastic theory and that computed from the measured reactions could be due

TABLE 26

ULTIMATE LOAD AND MAXIMUM THEORETICAL UNIT STRESS FOR UNSYMMETRICALLY LOADED ARCHES

Arch No.	Width of Rib in.	Point of Failure	Ultimate Load lb.	Section No.	Unit Stress at Ultimate Load by Elastic Theory in lb. per sq. in.
(1)	(2)	(3)	(4)	(5)	(6)
27-3	6⅛	Load-Point N4	111 436	N1 N2 N3 Center S3 S2 S1	3600 1616 2864 1716* 3009 2117 1226
27-4	6⅛	Section S3	135 917	N1 N2 N3 Center S3 S2 S1	4390 1971 3493 2093 3670* 2582 1495

*Section where failure occurred.

to the use of a horizontal thrust that was too small for loads below 90 000 lb. and one that was too large at greater loads. That is, Fig. 35 indicates that the divergency between the two load-thrust diagrams for arch 26-10 was due to a slight error in the measured value of the horizontal thrust.

Figure 24 shows the load-thrust lines for arches 27-3 and 27-4 to be straight lines having slopes slightly less than the slope of the theoretical line, that is, the measured value of the reaction is slightly greater than the theoretical value. Figures 36 and 37 show that, for the same arches, the value of the stress determined from the measured reactions does not agree with the theoretical value, and the differences could be accounted for by the use of a horizontal thrust that was too great. That is, the divergency between the diagrams, one showing the measured and the other the theoretical value of the thrust, appears to have been due to a slight error in the measured values for this arch also.

Because the moment in an arch is the small difference between two large moments of opposite signs, measuring the abutment reactions is not an accurate method of determining the stress in an arch, but measuring the stress is an extremely sensitive method of determining the horizontal thrust, and this sensitive method gives values that are in almost perfect agreement with the elastic theory.

TABLE 27

SECANT MODULUS OF CONCRETE IN ARCH AT STRESS OF 1000 LB. PER SQ. IN.

In 1 000 000 lb. per sq. in.

Arch No.	Section						
	N1	N2	N3	Center	S3	S2	S1
Symmetrically Loaded Arches							
26-1	2.38	2.60	2.39	2.18*	2.40	2.31	3.29
26-2	2.55	2.86	2.50	2.55	3.23	2.38*	2.48
26-3	2.63	2.17	2.19	1.15*	1.48	1.47	2.48
26-4	1.78	2.08	1.88	1.85*	2.14	2.00	1.95
26-5	3.00	2.44	2.43	2.00*	2.28	2.23	2.13
26-6	2.50	2.67	2.10	1.69*	2.48	2.25	2.28
26-10	2.00	1.62	1.86	1.44*	2.09	2.10	2.10
Unsymmetrically Loaded Arches							
27-3	2.54	1.98	1.54	1.34*	1.60	1.65	1.74
27-4	3.18	2.33	1.67	1.77	2.13*	2.38	2.63

*Section where failure occurred.

The relation between the strength of concrete as developed in an arch and the strength of the same concrete as developed in a 6-in. by 12-in. control cylinder is of vital importance because of its influence upon the unit stress to be used. However, an accurate comparison of the strength developed by concrete in an arch and in a cylinder is difficult because of the many factors affecting the strength of concrete. The large variations in the modulus of elasticity at the different sections of the same arch indicate that the concrete was not uniform in quality even though all parts of the arch were poured from the same mix and on the same day. For this reason the modulus of elasticity, as well as the ultimate unit stress, was used in studying the properties developed by the material in the arch and in the cylinder.

The theoretical unit stress corresponding to the ultimate load, at each of seven sections, is given in the right-hand column of Tables 25 and 26 for each of seven symmetrically loaded arches and two unsymmetrically loaded arches. The section at which failure occurred is indicated by an asterisk. Table 27 shows the modulus of elasticity of the concrete at the same sections. Not a single arch broke at a section of maximum stress, but seven of the nine arches each failed at a section for which the modulus of elasticity was a minimum. The exceptions were arches 26-4 and 27-4. Arch 26-4 broke at the center where the modulus of elasticity was 1 850 000 lb. per sq. in. instead

TABLE 28

COMPARISON OF UNIT STRESS DEVELOPED IN ARCH WITH UNIT STRESS DEVELOPED IN CONTROL CYLINDERS

Arch No.	Unit Stress		Secant Modulus at 1000 lb. per sq. in.			Section Having Max. E
	Arch	Cylinder	Arch at Section of Failure	Cylinder	At Section where it has Max. value	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
26-1..........	2479	2563	2.18	3.51	3.29	S1
26-2..........	2428	2590	2.38	4.08	3.23	S3
26-5..........	2394	2470	2.00	2.76	3.00	N1
26-6..........	2487	2990	1.69	2.94	2.67	N2
26-10.........	2307	1396*	1.44	1.41	2.10	S1
27-3..........	1716	3380	1.34	2.95	2.54	N1
27-4..........	3670	3575	2.13	4.00	3.18	N1

*This is the value reported but it is apparently in error.
The maximum unit stress occurred at Sections N1 and S1 for all arches of the 26 series and at Section N1 for those of the 27 series.

of at N3 where the modulus was one-half per cent less and the theoretical stress 10 per cent less than at the center. Arch 27-4 broke at S3 where E was 2 150 000 lb. per sq. in., instead of at the center where E was 15 per cent less and the theoretical stress 45 per cent less than at S3. That is, for the two arches that did not fail at the section of minimum E, the minimum E occurred at a section where the unit stress due to a given load was correspondingly lower than the modulus of elasticity.

The big variation in the modulus of elasticity of the concrete in an arch, and the fact that the arches in general broke at sections of small E rather than at sections of large unit stress, emphasize the necessity of placing concrete in the same manner over an entire specimen. The experimental arches were poured in a vertical position. The sides and bottom of the forms were in place when pouring began. Pouring started at the two ends simultaneously and the top of the form was added as pouring progressed. The concrete was hard to place near the ends and was more easily placed as the operations approached the center. The concrete hardest to place received the most attention and the consolidation was greater at the ends than at any other point, the degree of consolidation decreasing as the crown was approached. This is apparent from the values of E given in Table 27.

The unit strengths developed by the concrete in the arch and in the control cylinders are compared in columns 2 and 3 of Table 28. The unit stress for the arch is the theoretical stress at the section where failure occurred, corresponding to the ultimate load, and the unit stress for the cylinder is the average value obtained from tests of three cylinders for each arch, poured from batches that went into the structure. The moduli of elasticity of the concrete in the arch and in the cylinders are compared in columns 4 and 5 of Table 28. Only arches 6 in. or more in width were included in this comparison.

The strengths developed by the concrete in the arch and in the cylinder were almost the same for arches 26-1, 26-2, 26-5, and 27-4. For arch 26-10 the table shows a greater strength for the concrete in the arch than in the cylinder but the results of the cylinder tests appear to be in error. For arches 26-6 and 27-3 the unit stress developed in the arch was much less than that developed in the cylinder. The failure of arch 27-3 would appear to have been due to the poor concrete at the center ($E = 1\ 340\ 000$ lb. per sq. in.) since the arch developed a stress (theoretical) of 3000 lb. per sq. in. at S3. Arch 26-6 failed at the center where the modulus of elasticity was 1 690 000 lb. per sq. in., but developed a stress of 3428 lb. per sq. in. at N1 and S1 where the modulus of elasticity was 2 500 000 and 2 280 000 lb. per sq. in., respectively. If variation in the modulus of elasticity is accepted as an indication of variation in the strength of concrete, the failure of all low-strength arches can be attributed to weak concrete at the point of failure. Corresponding unit stresses in columns 2 and 3 of Table 28 differ greatly for some arches but when the unit stresses are studied in conjunction with the moduli of elasticity, the differences in strength would appear to be due more to differences in the quality of the concrete than to differences in the shape of the structure, arch or cylinder.

The results presented in Tables 25, 26, 27, and 28, apparently justify the belief that concrete will develop approximately the same strength in an arch and in a control cylinder providing the concrete is consolidated to the same degree in both structures.

By far the most important conclusions to be drawn from the data contained in Tables 25, 26, 27, and 28 are that the strength of an arch is determined by the strength of its weakest section and failure to consolidate the concrete at a single section may be as serious as a partial omission of cement, as the use of poor aggregate, or as the use of too much water. The arches used in these tests were more carefully poured than many concrete structures, yet the variations in the modulus of elasticity and presumably in the strength, are very large.

12. *Relation between Abutment Movements and Reactions.*—Tests were made upon arches 26-5, 26-6, 26-7, and 26-8 to determine the

TABLE 29

OBSERVED DATA BEARING UPON THE ELASTIC PROPERTIES OF ARCHES

Arch No.	Step	Load	Measured Movement				Measured Change of Reactions		
			x	θ_n*	θ_s*	Referred to Step No.	H	M_n*	M_s*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
26–5	1	24 400							
	2	25 000	0.154	−84	−25	1	2690	100.8	98.2
	3	24 420	0.006	−1778	−44	2	−610	−69.6	−48.3
	4	24 450	0.005	−1740	−1812	2	−1440	−104.7	−106.5
26–6	1	18 920							
	2	19 060	0.130	155	115	1	2470	85.8	87.2
	3	19 490	0.016	−1425	−31	2	−585	−51.2	−35.7
	4	18 920	0.014	−1500	−1627	2	−1968	−81.3	−84.8
26–6	1	22 440							
	2	22 320	0.202	−28	−31	1	3575	118.6	117.3
	3	22 420	0.099	−3820	+31	2	−1130	−64.5	−22.9
	4	22 165	0.088	−3440	−2820	2	−2945	−114.2	−110.8
26–7	1	21 045							
	2	20 895	0.146	81	−53	1	2860	97.2	100.5
	3	20 910	0.025	144	−2181	2	−943	−29.2	−60.7
	4	20 730	0.005	2326	−2413	2	−153	32.5	−27.8
26–7	1	41 085							
	2	41 150	0.109	−16	−44	1	2165	79.2	81.7
	3	41 145	0.027	169	−2209	2	−991	−29.4	−63.2
	4	40 875	0.007	2320	−2347	2	−202	35.9	−30.8
26–8	1	19 790							
	2	20 340	0.245	1438	519	1	3795	138.0	124.3
	3	20 350	0.034	−88	−3807	2	−1135	−33.7	−71.3
	4	20 350	0.007	3099	−3932	2	−45	16.3	−33.8
26–8	1	39 660							
	2	39 510	0.184	150	537	1	4281	+122.3	119.3
	3	39 490	0.024	162	−2381	2	−663	−33.6	−65.8
	4	39 620	0.003	2525	−2631	2	−211	24.0	−33.1

*θ_n is the rotation of the north abutment, θ_s that of the south one, likewise M_n is the moment at the north abutment and M_s that at the south one. Rotations are in 0.000001 radians; forces are in lb., and moments in 1000 in. lb.

relation between abutment movements and changes in the reactions. The steps involved in making these tests are described in the following paragraphs.

Step 1. The arch was loaded with a symmetrical load, the abutments being restrained, and all reactions, horizontal thrust, vertical reactions, and moments at the springing, were recorded.

Step 2. The abutments were spread, the abutment rotation and the change in load being kept as small as possible.

TABLE 29 (CONCLUDED)

OBSERVED DATA BEARING UPON THE ELASTIC PROPERTIES OF ARCHES

Arch No.	Step	Correction for Change in Load			Corrected Change of Reaction		
		H	M_n	M_s	H	M_n	M_s
(1)	(2)	(11)	(12)	(13)	(14)	(15)	(16)
26–5	1						
	2	−324	+0.3	+0.3	2366	101.1	98.5
	3	313	−0.3	−0.3	−297	−69.9	−48.6
	4	297	−0.3	−0.3	−1143	−105.0	−106.8
26–6	1						
	2	−32	+0.1	+0.1	2438	85.9	87.3
	3	−276	+0.2	+0.2	−861	−50.0	−35.5
	4	32	−0.1	−0.1	−1936	−81.4	−84.9
26–6	1						
	2	64	0.0	0.0	3539	118.6	117.3
	3	−55	0.0	0.0	−1185	−64.5	−22.9
	4	136	0.0	0.0	−2809	−114.2	−110.8
26–7	1						
	2	81	0.0	0.0	2941	97.2	100.5
	3	−11	0.0	0.0	−954	−29.2	−60.7
	4	86	0.0	0.0	−67	32.5	−27.8
26–7	1						
	2	−35	0.0	0.0	2130	79.2	81.7
	3	2	0.0	0.0	−989	−29.4	−63.2
	4	148	−0.1	−0.1	−54	35.8	−30.9
26–8	1						
	2	−297	0.3	0.3	3498	138.3	124.6
	3	−5	0.0	0.0	−1140	−33.7	−71.3
	4	−5	0.0	0.0	−50	16.3	−33.8
26–8	1						
	2	80	0.0	0.0	4361	+122.3	119.3
	3	10	0.0	0.0	−653	−33.6	−65.8
	4	−60	0.0	0.0	−271	24.0	−33.1

Step 3. The north abutment was rotated, the span, load, and angular positions of the south abutment being kept as nearly constant as possible.

Step 4. The south abutment was rotated, the span, load, and angular position of the north abutment being kept as nearly constant as possible.

The observations that were made gave data from which the following quantities were computed:

(a) Thrust and moment at springing due to unit change in span.

(b) Thrust, moment at near springing, and moment at far springing due to unit rotation of the north abutment.

(c) Thrust, moment at near springing, and moment at far springing due to unit rotation of the south abutment.

Table 29 shows the observations that were made on the four arches enumerated above. Observations were made on all of the arches at two different loads but the observations on 26-5 at the lower load, which consisted only of the weight of the rib and apparatus, were so obviously in error that they are not reported. The movements of the abutments are given in columns 4, 5, and 6 of Table 29, and the observed changes in the reactions are given in columns 8, 9, and 10. As noted in column 7, the changes for Step 2, as shown in column 2, are based upon conditions for Step 1, and the changes for Steps 3 and 4 are based upon conditions for Step 2. The load for each step is given in column 3. It was not practicable to maintain the load exactly constant, and the theoretical changes in the reactions due to the slight changes in the load, based upon Table 6, are presented as corrections in columns 11, 12, and 13. The corrected changes in the reactions, and the changes due to abutment displacements only, are given in columns 14, 15, and 16.

There were three sets of measured abutment movements for each arch, and there was a measured value of H, *of* M_n, and of M_s for each movement. The relations between movements and changes in reactions may, therefore, be treated as groups of three simultaneous equations, one group for each test, and each group can be solved for the unknowns that it contains, thereby obtaining the values of H, M_n, and M_s due to a unit spread of the abutments, a unit rotation of the north abutment, and a unit rotation of the south abutment. The results thus obtained from the observed data contained in Table 29 are given in Table 30.

The arch was symmetrical about the vertical centerline so that M_n and M_s due to unit spread of the abutments should be equal. The values of these quantities are given in adjacent columns and their average in column 5. Likewise M_n due to θ_n and M_s due to θ_s should be equal. Their average is given in column 10. The moment M_s due to θ_n and M_n due to θ_s should be equal. Their average is given in column 11. The thrusts due to θ_n and θ_s should be equal. Their average is given in column 15.

The measured and the computed values of the reactions due to unit displacement of the abutments are compared in Table 31. The measured value of the reactions, given in columns 3, 6, 9, 12, and 15, were taken from columns 5, 10, 11, 12, and 15, respectively, of Table 30. The computed values of the reactions were taken from Table 4

TABLE 30

MEASURED REACTIONS DUE TO ABUTMENT DISPLACEMENTS

Arch No.	Load	Moment Due to Unit Movement of Abutments									Thrust Due to Unit Movement of Abutments			
		$x = 0.10$ in.			$\theta_n = 0.001$ radian		$\theta_s = 0.001$ radian		Av. M at End Rotated	Av. M at End Not Rotated	$x = 0.10$ in.	$\theta_n = 0.001$ radian	$\theta_s = 0.001$ radian	Av. θ_n and θ_s
		M_n	M_s	Av.	M_n	M_s	M_n	M_s						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
26-5	24 400	68.22	66.07	67.15	41.11	28.75	20.35	33.16	37.14	24.55	1555	207.8	474.2	341.00
26-6	18 920	59.63	60.90	60.27	41.40	31.10	16.98	28.72	35.06	24.04	1727	784.8	615.0	700.00
26-6	22 440	59.46	58.85	59.16	32.46	21.50	19.46	31.43	31.95	20.48	1772	774.0	591.9	683.00
26-7	21 045	65.39	68.79	67.09	36.74	25.36	23.31	37.39	37.07	24.34	2003	665.7	711.0	688.35
26-7	41 085	74.25	76.93	75.59	38.82	24.80	25.36	39.91	39.37	25.08	1992	582.4	735.6	659.00
26-8	19 790	41.90	37.02	39.46	20.40	15.76	12.17	21.67	21.04	13.97	883	448.8	385.8	417.30
26-8	39 660	57.41	52.86	55.14	32.11	23.30	22.08	34.55	33.33	22.69	2184	417.0	522.7	494.90

Loads and thrusts are in lb., moments in 1000 in.-lb.

and are based upon a value of E for the concrete of 2 500 000 lb. per sq. in. The ratio of the measured and the computed values are given in columns 4, 7, 10, 13, and 16.

The term "composite E" has been used. It may be defined as the value of E for the concrete that would make the computed values of the reactions agree with the measured values. The values of the composite E, as determined from the reactions resulting from the various abutment displacements are given in columns 5, 8, 11, 14, and 17. The reaction due to a unit abutment displacement is proportional to IE and, if the section of the rib were homogeneous, would also be proportional to E. But, since the rib contains reinforcing steel, the value of I changes with E, and the reactions are not exactly proportional to E. Computations with E at 2 500 000 lb. per sq. in. and E at 3 000 000 lb. per sq. in. show that the relative change in the reactions due to abutment movements averages approximately 1/1.05 times the relative change in E. The values of composite E, contained in Table 31, were obtained in the following manner: If the ratio $\frac{M_m}{M_c}$* exceeds unity, the composite $E = \frac{M_m}{M_c} \times 1.05 \times 2\ 500\ 000$ lb. per sq. in. If the ratio is less than unity, the composite $E = \left[1 - 1.05\left(1 - \frac{M_m}{M_c}\right)\right] 2\ 500\ 000$ lb. per sq. in.

The measured and the computed values of the thrust differed greatly for arch 26-5 and for arch 26-8 when the latter carried a load of 19 790 lb., whereas, for all other arches and loads, the measured and the computed values are in fair agreement. The data contained in lines 1 and 6 were therefore omitted in computing the averages given at the bottom of Table 31.

The value of the composite E was determined from five relations as follows:

(1) The moment at the springing due to a change in span
(2) The horizontal thrust due to a change in span
(3) The horizontal thrust due to the rotation of an abutment
(4) The moment at a springing due to the rotation of the adjacent abutment
(5) The moment at a springing due to the rotation of the opposite abutment

The values of the composite E are presented in Tables 32 and 33. In the former the values from a given relation are averaged for the

*M_m is measured and M_c is computed moment.

five tests, in the latter the values from the five relations are averaged for each test. As given in Table 32, the values of E determined from the moment at the springing due to a change in span, and from the thrust due to a rotation of an abutment, are both nearly equal to 2 500 000 lb. per sq. in. The value of E obtained from the moment adjacent to a rotated abutment is less than 2 500 000 lb. per sq. in., and the values of E obtained from the thrust due to a change in span and from the moment at the springing opposite to the abutment rotated are larger than 2 500 000 lb. per sq. in. From Table 33 the composite E for arch 26-6 was 2 758 000 lb. per sq. in. at a load of 18 920 lb. and 2 605 000 lb. per sq. in. at a load of 22 440 lb. The average unit stress over the entire arch was approximately 240 lb. per sq. in. at the former, and 280 lb. per sq. in. at the latter load. The E of the same arch, the average of the value determined from strain-gage readings at the top and bottom of seven sections, as given in Table 27 was 2 280 000 lb. per sq. in. at a unit stress of 1000 lb. per sq. in. The composite E of arch 26-7 was 2 958 000 and 3 080 000 lb. per sq. in. at 21 045-lb. and 41 085-lb. loads, corresponding to average unit stresses of 267 and 516 lb. per sq. in., respectively. The composite E of arch 26-8 was 2 614 000 lb. per sq. in. at a 39 660-lb. load, corresponding to an average unit stress of 500 lb. per sq. in. The modulus of elasticity of the concrete in arches 26-7 and 26-8 was not determined from strain-gage readings but the average value for all arches given in Table 27 was 2 225 000 lb. per sq. in.

The experimental data would appear to justify the statement that the composite E determined from measured abutment reactions due to abutment movements agreed closely with the average E determined from strain-gage readings at the top and bottom of seven sections of the arch.

The computations for determining the effect of abutment movements were based upon the assumptions that E for the concrete has the same value at all sections and at all loads and, in determining the moment of inertia of a section, that the concrete resists tension. The computations for the horizontal thrust and for the theoretical stress due to load were based upon the same assumptions. The close agreement between theoretical and measured values, when the former were computed on this basis, would appear to justify the following practice.

In analyzing an arch to determine the thrust and moment at the various sections, E may be assumed to have the same value at all sections and at all loads, and the moment of inertia of a section may

TABLE 31

COMPARISON OF OBSERVED AND COMPUTED VALUES OF REACTIONS DUE TO ABUTMENT DISPLACEMENTS

Arch No.	Load	Moment at Springing Due to Movement of Abutment								
		x = 0.10 in.			θ = 0.001 radian					
		Measured Value M_m	Ratio $\frac{M_m}{M_c*}$	Composite E, in 10^6	Measured Value at End Rotated M_m	Ratio $\frac{M_m}{M_c†}$	Composite E, in 10^6	Measured Value at End Not Rotated M_m	Ratio $\frac{M_m}{M_c‡}$	Composite E, in 10^6
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
26-5........	24 400	67.15	1.057	2.774	37.14	0.996	2.490	24.55	1.342	3.523
26-6........	18 900	60.27	0.948	2.363	35.06	0.940	2.343	24.04	1.314	3.449
26-6........	22 440	59.16	0.931	2.319	31.95	0.857	2.125	20.48	1.119	2.937
26-7........	21 045	67.09	1.056	2.772	37.07	0.994	2.485	24.34	1.330	3.491
26-7........	41 085	75.59	1.190	3.124	39.37	1.056	2.772	25.08	1.371	3.599
26-8........	19 790	39.46	0.621	1.505	21.04	0.564	1.355	13.97	0.764	1.880
26-8........	39 660	55.14	0.868	2.153	33.33	0.894	2.223	22.69	1.240	3.255
Average.....		63.45	0.9986	2.546	35.36	0.948	2.390	23.33	1.275	3.346

Moments are in 1000 in. lb.; thrusts are in lb. Computed values are based upon E = 2 500 000 lb. per sq. in.
*M_c = 63.543
†M_c = 37.279
‡M_c = 18.297
The 1st and 6th lines were omitted in computing averages.
The subscripts m and c indicate measured and computed values.
Composite E is the value of E that would make the measured and computed values equal. (See text page 88).

TABLE 31 (CONCLUDED)

COMPARISON OF OBSERVED AND COMPUTED VALUES OF REACTIONS DUE TO ABUTMENT DISPLACEMENTS

Arch No.	Load	Thrust Due to Movement of Abutment					
		x = 0.10 in.			θ = 0.001 radian		
		Measured Value H_m	Ratio $\frac{H_m}{H_c{}^*}$	Composite E, in 10^6	Measured Value H_m	Ratio $\frac{H_m}{H_c{}^\dagger}$	Composite E, in 10^6
(1)	(2)	(12)	(13)	(14)	(15)	(16)	(17)
26-5	24 400	1 555	0.970	2.421	341.0	0.521	1.242
26-6	18 920	1 727	1.077	2.827	700.0	1.070	2.809
26-6	22 440	1 772	1.105	2.901	683.0	1.044	2.741
26-7	21 045	2 003	1.250	3.281	688.4	1.052	2.762
26-7	41 085	1 992	1.243	3.263	659.0	1.007	2.643
26-8	19 790	883	0.551	1.323	417.3	0.638	1.603
26-8	39 660	2 184	1.362	3.575	494.9	0.757	1.863
Average		1 936	1.207	3.169	645.1	0.986	2.564

Moments are in 1000 in. lb.; thrusts are in lb. Computed values are based upon E = 2 500 000.
*H_c = 1 603
†H_c = 654
The 1st and 6th lines were omitted in computing averages.
The subscripts m and c indicate measured and computed values.
Composite E is the value of E that would make the measured and computed values equal. (See text page 88).

TABLE 32

VALUE OF COMPOSITE E OBTAINED FROM VARIOUS RELATIONS

Relation from which E was Determined	Maximum E	Minimum E	Average of Five Tests E
(1)	(2)	(3)	(4)
(1) Moment at Springing Due to Change in Span	3 124 000	2 153 000	2 546 000
(2) Horizontal Thrust Due to Change in Span	3 575 000	2 827 000	3 169 000
(3) Horizontal Thrust Due to Rotation of an Abutment	2 809 000	1 863 000	2 564 000
(4) Moment at a Springing Due to the Rotation of the Adjacent Abutment	2 772 000	2 125 000	2 390 000
(5) Moment at a Springing Due to the Rotation of the Opposite Abutment	3 599 000	2 937 000	3 346 000

TABLE 33

AVERAGE VALUE OF COMPOSITE E FOR VARIOUS ARCHES

Arch No.	Load	Composite E, Average of Values Obtained from Five Relations	Strength of Concrete as Determined from Tests of Cylinders
26-6	18 920	2 758 000	2918
26-6	22 440	2 605 000	2918
26-7	21 045	2 958 000	3112
26-7	41 085	3 080 000	3112
26-8	39 660	2 614 000	2452

be based upon the assumption that the concrete resists tension. It does not necessarily follow, however, that the moment of inertia used in computing the unit stress on a section due to a specified moment should be based upon the same assumptions.

13. *Value of Spiral Reinforcing.*—Arches 26-6 and 26-10 were identical with 26-5 except that they were reinforced with a spiral approximately 5 in. in diameter made of ¼-in. rounds and having a pitch of 1 inch. Arch 26-2 was the same as 26-5 except that the longitudinal reinforcing was $\frac{7}{16}$-in. rounds for the former, and ½-in. rounds for the latter. The ultimate loads for 26-2 and 26-5, the two *not having* spiral reinforcing, were 153 110 and 154 470 lb., respectively; the corresponding loads for 26-6 and 26-10, the two *having*

spiral reinforcing, were 159 450 and 147 860 lb., respectively. The ultimate loads on the control cylinders, the average of three for each arch, were 2595 and 2470 lb. per sq. in. for 26-2 and 26-5, and 2990 and 1396 lb. per sq. in. for 26-6 and 26-10. This evidence, considered by itself, would indicate that the spiral reinforcement had not affected the strength of the arch. When the secant modulus at the section of failure is considered, however, the case is not so clear, the moduli for 26-2, 26-5, 26-6, and 26-10, as given in Table 27, being 2 380 000, 2 000 000, 1 690 000 and 1 440 000 lb. per sq. in., respectively. If the secant modulus is accepted as a criterion of strength, arches 26-2 and 26-5 had better concrete at the section of failure than arches 26-6 and 26-10. The results of the tests are reported as interesting information but they cannot be looked upon as conclusive evidence.

Spiral reinforcement increases the strength of short thick columns, but it does not materially increase the strength of long slender ones, nor does it increase the strength of a member subjected to flexure. An arch rib is a long slender compression member subjected to flexure and, reasoning from experience with columns and flexural members, there is no reason to expect the spiral reinforcing to materially increase its strength. The manner of failure of these arches, a buckling that increased the flexural stress as soon as the concrete outside of the spiral had spalled (see Fig. 16), was more convincing evidence than a comparison of the ultimate strength that spiral reinforcement does not increase the strength of slender arch ribs.

14. *Relation between Moment at Springing and Rotation of Abutments.*—Arches 26-7, 26-8, and 26-9 were tested to determine the relation between the moment at the springing and the rotation of the abutments. All arches were 6⅛ in. wide, and were reinforced with four ½-in. round longitudinal bars. Arch 26-8 also contained a 5-in. spiral reinforcement made of ¼-in. rounds and having a 1-in. pitch.

The manner of testing was as follows: The arch was loaded with a symmetrical load of 50 000 lb., the abutments remaining as nearly stationary as practicable. Then, with the total load and span constant, the abutments were rotated by manipulating the jacks under the outer ends of the abutment levers, and the magnitude of the moment was obtained from the loads on the jacks, measured with the portable scales. In making a test the abutments were always either both tipped in or both tipped out, the change in moment being the same for both abutments.

The test of arch 26-7 began on April 19. The arch having been loaded to 50 000 lb., the abutments were subjected to a negative moment by reducing the load on the jacks until they were free. This being the limit of negative moment, a positive moment was produced by extending the jacks until the abutments would no longer rotate with the abutment levers L4. The test was then discontinued and the arch reset, this time in such a way that a greater moment could be produced at the springing. The test was resumed April 23, and the arch was finally broken that day by increasing the moment at the springing, the total load on the arch throughout this procedure remaining at approximately 50 000 lb. Failure occurred at the north springing, the character of the failure being illustrated in Fig. 18.

The relation between the moment at the springing and the rotation of the abutments is shown in Fig. 52. The full lines represent the movement of the north abutment, and the broken lines the movement of the south abutment. An effort was made to rotate the abutments without changing the span of the arch. As a matter of fact the span did change slightly, and the measured moments were corrected for the moment caused by the small change in span. The dot-and-dash line represents the theoretical relation between moment and rotation for a value of E of 2 500 000 lb. per sq. in. The diagrams show that the measured and the theoretical rotations agree for the initial loading. After the arch had been reset the measured rotation increased more rapidly than the theoretical. The south abutment did not rotate as much as the north, indicating a larger value of E for the concrete for the former than for the latter. The strain-gage readings before and after the application of the 50 000-lb. load also show a greater strain at the north than at the south end of the arch. The arch failed at a load of 50 000 lb. when each abutment was subjected to a moment of 294 000 in. lb. The rotation was roughly 1.8 times as great as the rotation corresponding to this moment, as computed by the elastic theory.

The test of arch 26-8 was completed in a single day. A load of 50 000 lb. was applied with fixed abutments, and then the moment at the springing was increased without appreciable change in load or span until the arch failed at the north springing. The character of the failure is shown in Fig. 19.

The relation between the rotation of the abutments and the moment at the springing is shown in Fig. 52. The measured moments were corrected by the elastic theory for the slight change in the span that occurred during the test. The north and south abutments behaved very much alike, and the measured and theoretical values of the movement agreed closely up to a moment of 145 000 in. lb. As

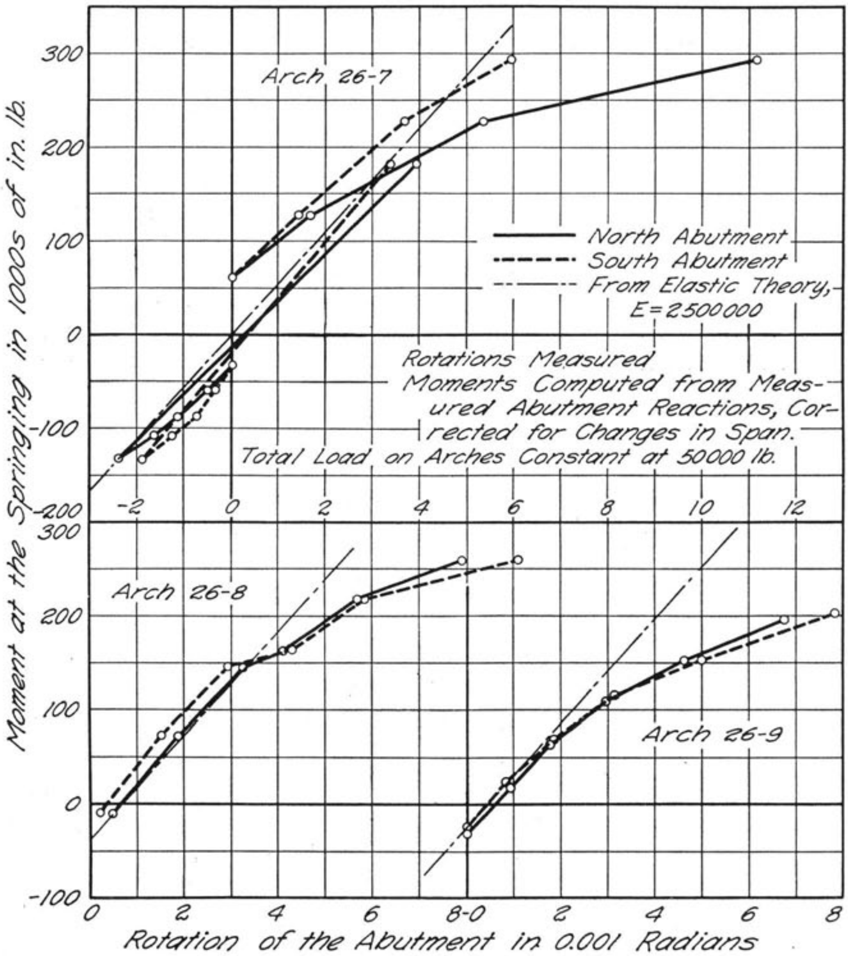


FIG. 52. RELATION BETWEEN ROTATION OF ABUTMENT AND MOMENT AT SPRINGING, ARCHES 26-7, 26-8, AND 26-9

the moment increased beyond this value the rotation increased quite rapidly. A small crack was detected on the intrados at the north springing at a moment of 144 000 in. lb. This and other cracks developed as the moment was increased. The arch failed at a moment of 280 000 in. lb., and the rotation at the ultimate was about 1.7 times the rotation given by the elastic theory for the same moment.

Arch 26-9 was tested in a single day and in the same manner as 26-8. The relation between the moment and rotation is shown in

TABLE 34

ULTIMATE UNIT STRESS DEVELOPED AT A SECTION OF THE ARCH RIB SUBJECTED TO A LARGE MOMENT

Arch No.	Load on Arch	Ultimate Moment at Each Springing	Maximum Compression at Ultimate Load on the Basis that:		Strength of Control Cylinders
			Concrete takes tension	Concrete does not take tension	
26-7	50 000	294 000	2409	2935	3040
26-8	50 000	280 000	2314	2840	2557
26-9	50 000	210 000	1859	2170	1511*

Loads are in lb., moments in in.-lb., and stress in lb. per sq. in.
*This value was reported, but it is apparently in error.

Fig. 52. The two ends of the arch behaved in the same manner, and at small moments the measured and theoretical rotations were in very close agreement. The arch failed at a moment of about 210 000 in. lb., and the rotation at failure was practically 1.8 times the theoretical value. Cracks were detected on the intrados of both springings at a moment of about 115 000 in. lb., and these widened and others formed as the moment increased. The arch failed at the south springing, the character of the failure being shown in Fig. 20.

The tests of these three arches indicate that the measured rotation of an abutment due to moment agrees closely with the theoretical value until cracks are formed in the rib, and then the former gradually becomes greater than the latter. When the arch was subjected to a load equal to about one-third of the ultimate for the arch with fixed abutments, failure at the springing occurred under a moment at the springing of approximately 294 000, 280 000, and 210 000 in. lb., respectively. The corresponding rotations were approximately 0.01 radian. The magnitude of this angle is best visualized by considering the corresponding tipping of a pier. If a pier base is 20 ft. wide, a rotation of 0.01 radian would require one side of the base to rise and the other to fall a distance of 1.2 in.

The unit stress at the springing due to the load and the rotation of the abutment computed from the measured abutment reactions are given in Table 34. The computations were made on the basis of the two assumptions that the concrete resists tension and that it does not resist tension. The stresses computed upon these two assumptions are compared with the unit stress developed by the

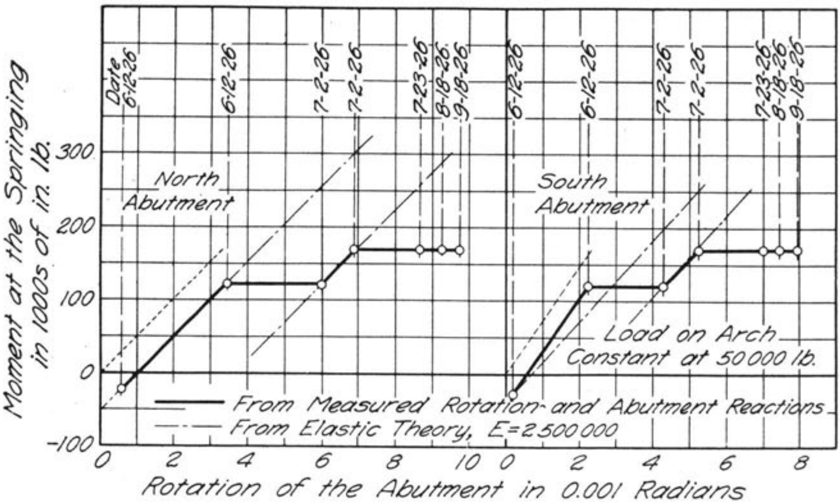


FIG. 53. INFLUENCE OF TIME YIELD UPON RELATION BETWEEN ROTATION OF ABUTMENT AND MOMENT AT SPRINGING

control cylinder in the last three columns of the table. The probable error in the cylinders for arch 26-9 makes the test of this arch of little value. The results of the tests of arches 26-7 and 26-8 show that the ultimate unit stress developed at a section of the arch subjected to a large flexure did not differ materially from the strength developed by the concrete in 6-in. by 12-in. control cylinders.

15. *Effect of Time Yield upon Rotation of Abutments.*—Arch 26-11 was tested to determine the effect of time yield upon the rotation of the abutments due to moment. The method of testing was as follows: A load of 50 000 lb. was applied on June 12 without appreciable movement of the abutments. The jacks under the outer ends of the abutment levers were then extended until the moment at the springing was approximately 122 000 in. lb., the span of the arch remaining practically constant. During the period from June 12 until July 2 the jacks and the testing machine were adjusted each morning except Sunday, so that the total load on the arch would be 50 000 lb., and the moment on each abutment would be 122 000 in. lb. On the latter date the moment was increased to 170 000 in. lb., the total load remaining constant at 50 000 lb. The machines were adjusted each morning to these loads and moments until August 2; they were not adjusted after this date until August 18, when they had fallen off

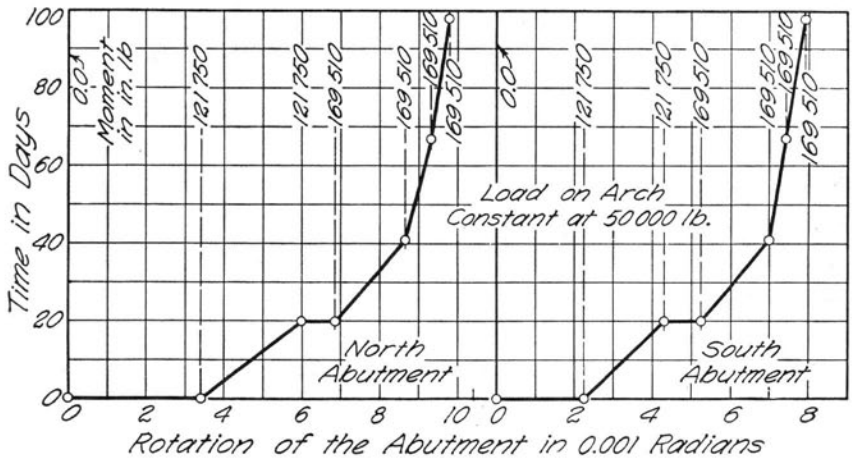


FIG. 54. INFLUENCE OF TIME YIELD UPON ROTATION OF ABUTMENTS

about 5 per cent. The load and moments were brought up to 50 000 lb. and 170 000 in. lb., respectively, and, without further adjustment, they again fell off about 5 per cent by Sept. 2. The loads were adjusted each morning from Sept. 2 to Sept. 18.

The relation between the moment and the rotation is shown in Fig. 53 for the north and for the south abutment. The horizontal portion of this diagram shows the rotation due to time yield under constant load and moment. The dash-and-dot line shows the relation between moment and instantaneous rotation as given by the elastic theory. The rotations that accompany an increase in the moment agree almost perfectly with the theoretical value, with E taken at 2 500 000 lb. per sq. in. The total rotation from June 12 to Sept. 18 is about three times the theoretical instantaneous rotation for the same loads and moments.

The relation between rotation and time is shown by the diagrams of Fig. 54 for the north and for the south abutment. The rotation is seen to increase with the lapse of time without an increase in the moment, but the rate of increase is much less at the end than at the beginning of the period.

16. *Effect of Slenderness of an Arch Rib upon its Strength.*—Arches 26-1, 26-2, 26-3, and 26-4 formed a series that was planned to determine the effect of the slenderness ratio of a rib upon its strength. The arches were identical in profile and identically loaded

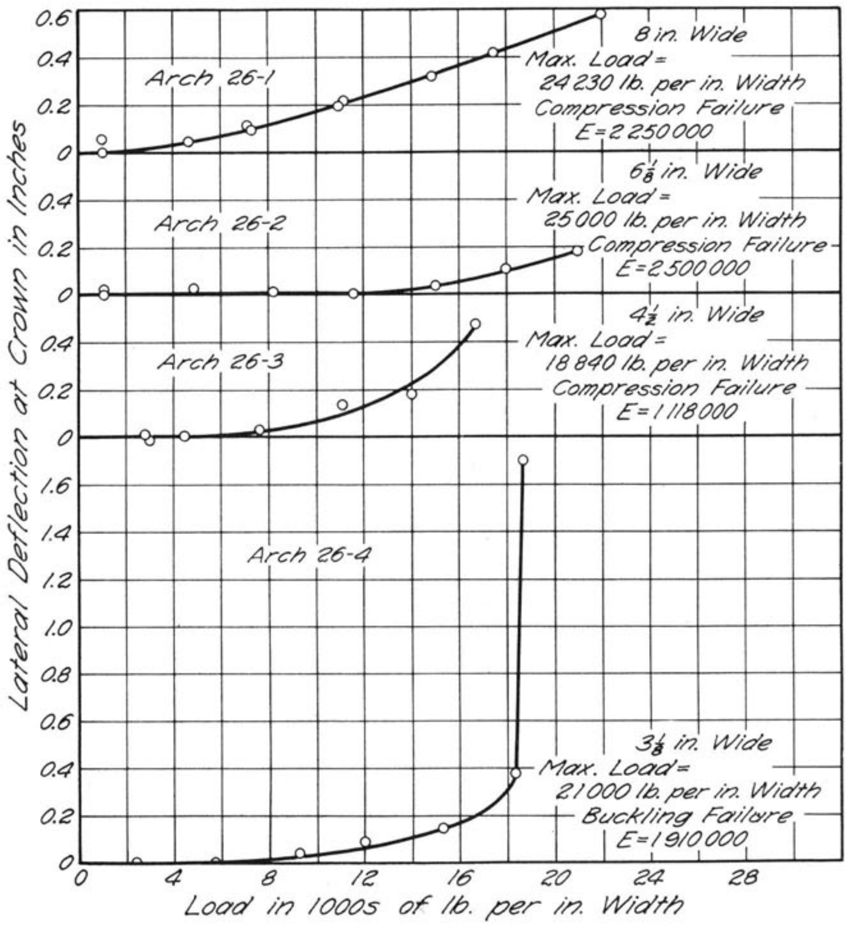


FIG. 55. RELATION BETWEEN LOAD AND LATERAL DEFLECTION AT CROWN

but varied in width, the widths being 8 in., 6⅛ in., 4½ in., and 3⅛ in., respectively. The percentage of reinforcing was the same for all arches.

Measurements were made to determine the lateral deflection at the crown as the load increased. The relation between the load per inch width of rib and the lateral deflection is shown in Fig. 55. Arches 26-1 and 26-2 deflected somewhat at the crown, but there was no evidence of impending buckling. Arch 26-3 did not fail by buckling, but the shape of the diagram indicates that buckling was impending. Arch 26-4 had a typical buckling failure.

TABLE 35

EFFECT OF SLENDERNESS RATIO UPON STRENGTH OF AN ARCH RIB

Arch No.	Width in in.	Slenderness Ratio	Ultimate Load per Inch Width, lb.	Secant Modulus at 1000 lb. per sq. in. Stress at Section of Failure
26-1	8	30	24 230	2 180 000
26-2	6⅛	39	25 000	2 380 000
26-3	4½	53	18 840	1 150 000
26-4	3⅛	76	21 000*	1 850 000

*Buckling began at 18 700 lb. per inch width of rib and if this load had been retained, failure would probably have eventually resulted without further increase in the load.

The slenderness ratio and the strength per inch width of rib are shown in Table 35. The strength of 26-3 is quite low but, as explained in Section 11, this would appear to be due to poor concrete at the crown as indicated by the low secant modulus. Arch 26-4 with a slenderness ratio of 76 was weakened by its tendency to buckle. The strength of arches 26-1 and 26-2 does not seem to have been affected by any tendency to buckle. The results of these tests would apparently justify the conclusion that ribs having a ratio of unsupported length to width of 30 are not weakened by a tendency to buckle so long as they are not subjected to the action of lateral forces. Practical considerations should prevent the use of ribs having a slenderness ratio greater than 30.

VI. CONCLUSIONS

17. *Summary of Results.*—An arch is affected by so many factors that final conclusions cannot be obtained from a few tests. A study of the results, however, appear to justify the following statements:

(1) The measured and the theoretical values of the reactions due to load, both moment and thrust, are in close agreement.

(2) The measured and the theoretical values of the reactions due to abutment displacement are in close agreement for these arches if the theoretical value is based upon a modulus of elasticity equal to the average of the values obtained from strain-gage readings at seven sections of the rib.

(3) The measured and the theoretical values of the unit stress due to load agree closely.

(4) The unit stress developed by the concrete in the arches was approximately the same as the stress developed by the same concrete in 6-in. by 12-in. control cylinders.

(5) A considerable accidental local variation in the moment of inertia or the modulus of elasticity does not materially affect the behavior of an arch rib.*

(6) In analyzing the arch to determine the thrust and moment at various sections, the modulus of elasticity of the concrete may be assumed to have the same value at all loads and at all sections; and the moment of inertia of a section may be based upon the assumption that the concrete resists tension. The errors in these assumptions will not materially affect the value of the moment and thrust determined by an analysis based upon the assumptions. It does not follow, however, that the stress at a given section due to a specific moment should be based upon the same assumption.

(7) If an arch rib is not subjected to lateral forces its strength does not appear to be affected by its slenderness, so long as the ratio of the unsupported length of the arch axis to the width of the rib does not exceed 30.

*This statement is based upon a theoretical analysis made by Hardy Cross. See "Dependability of the Theory of Concrete Arches," Univ. of Ill. Eng. Exp. Sta. Bul. 203.

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